Evaluation and Enumeration Problems for Regular Path Queries

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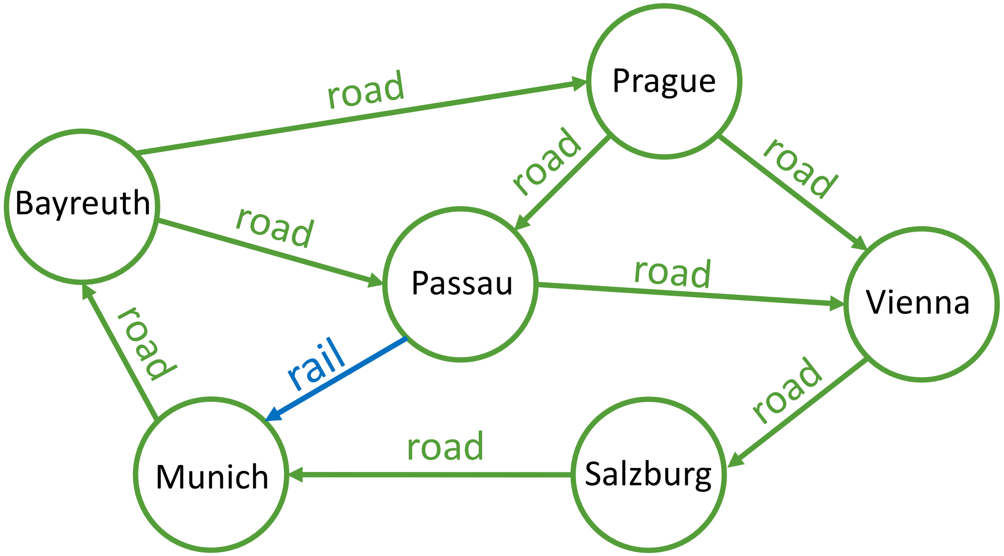
University of Bayreuth

Practice

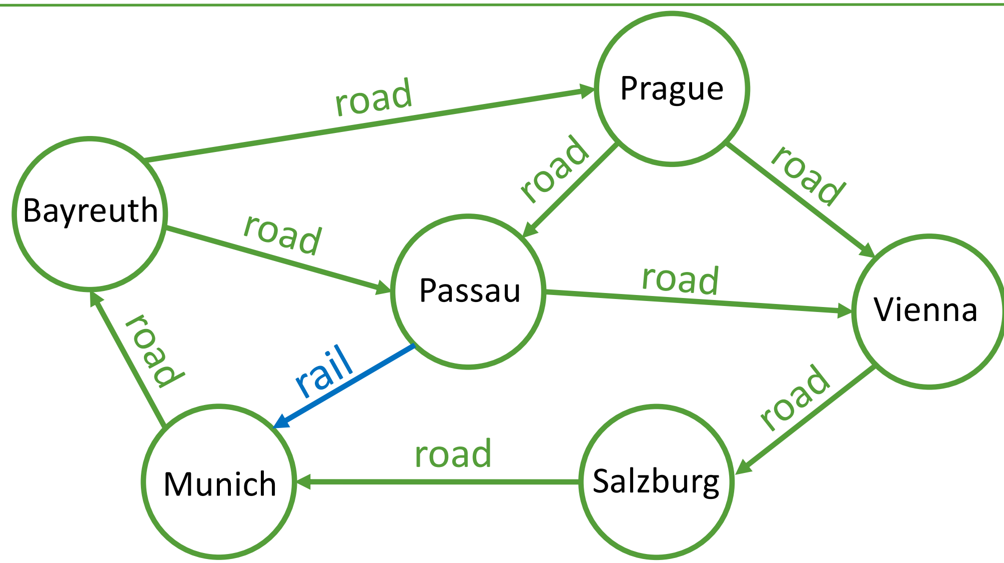
QUERYING PATHS IN GRAPH DATABASES

Graph Database



Node- and Edge-labeled directed graph

Warm Up Question



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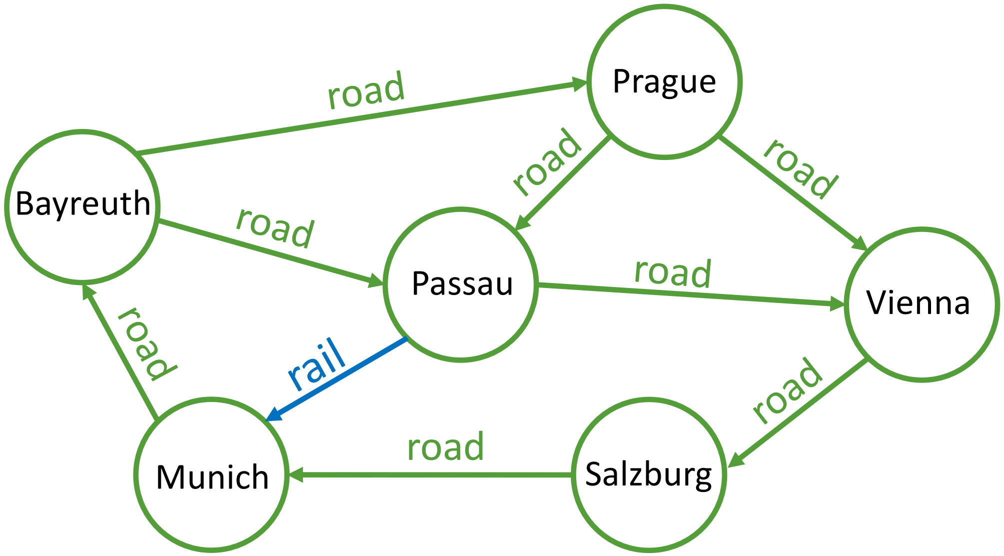

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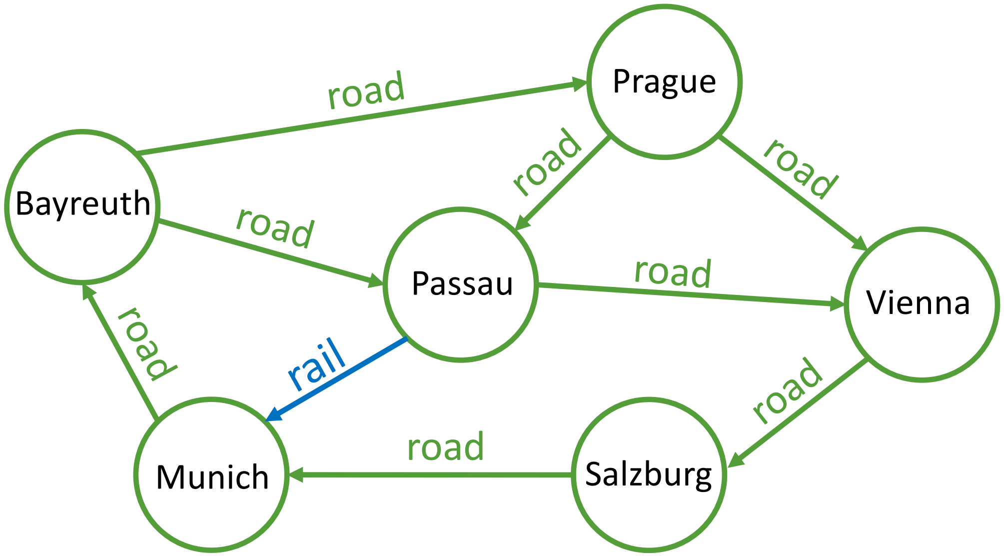

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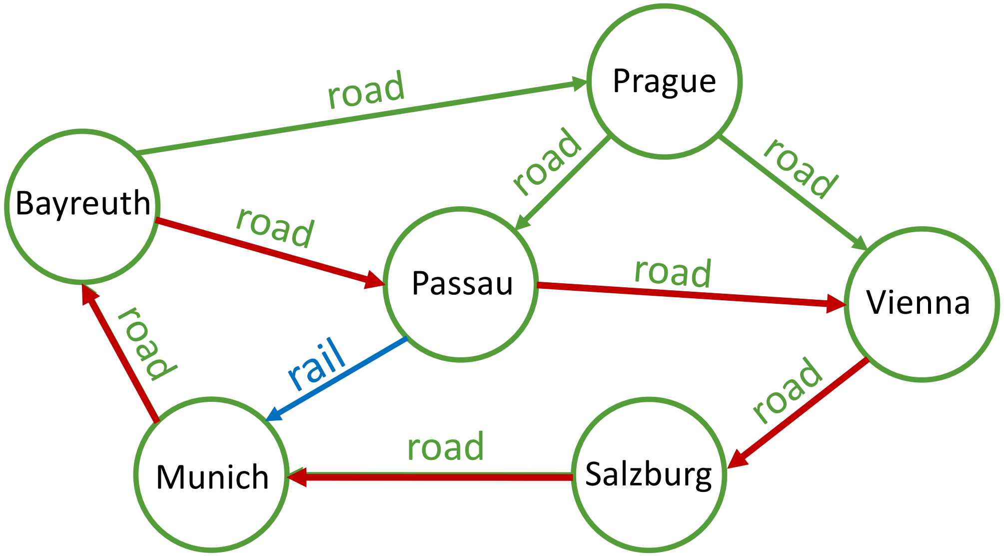

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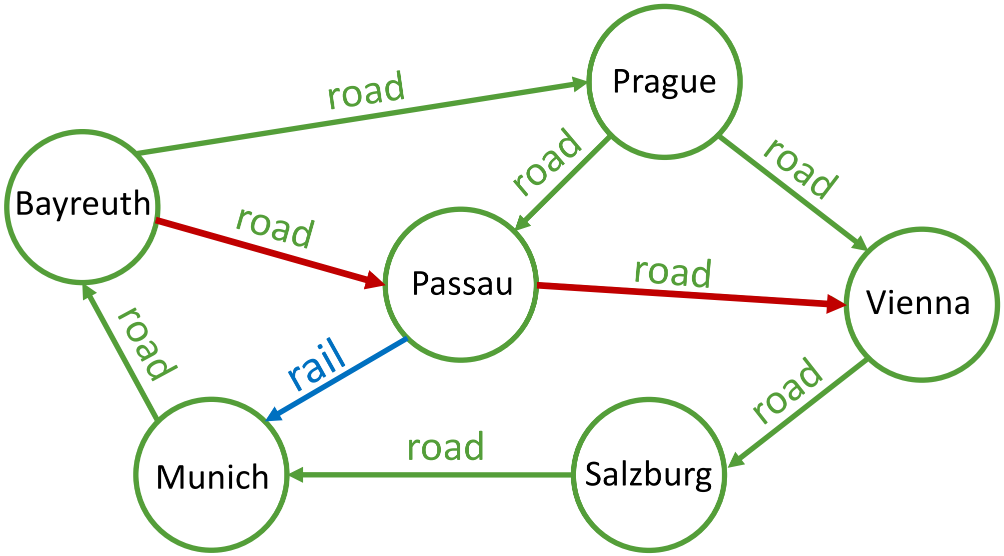

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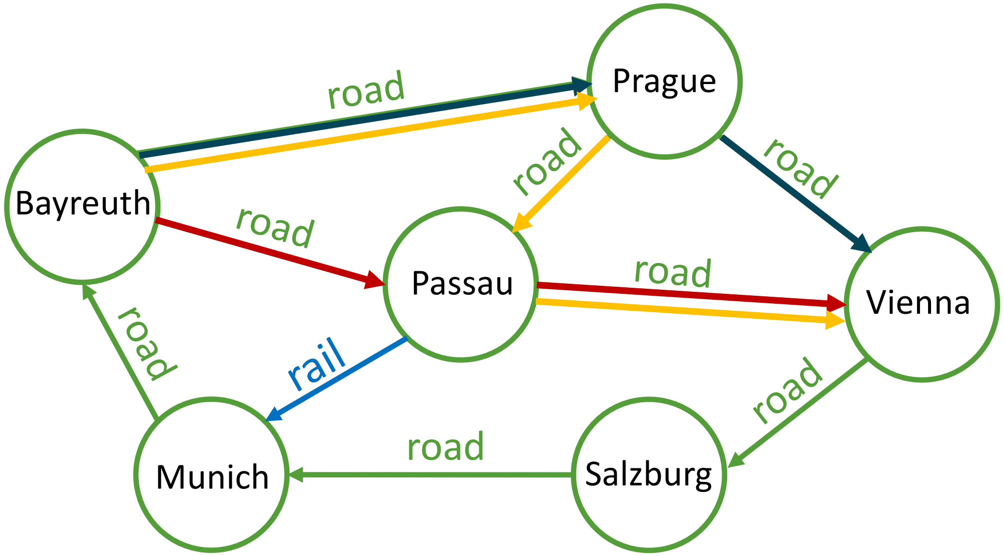

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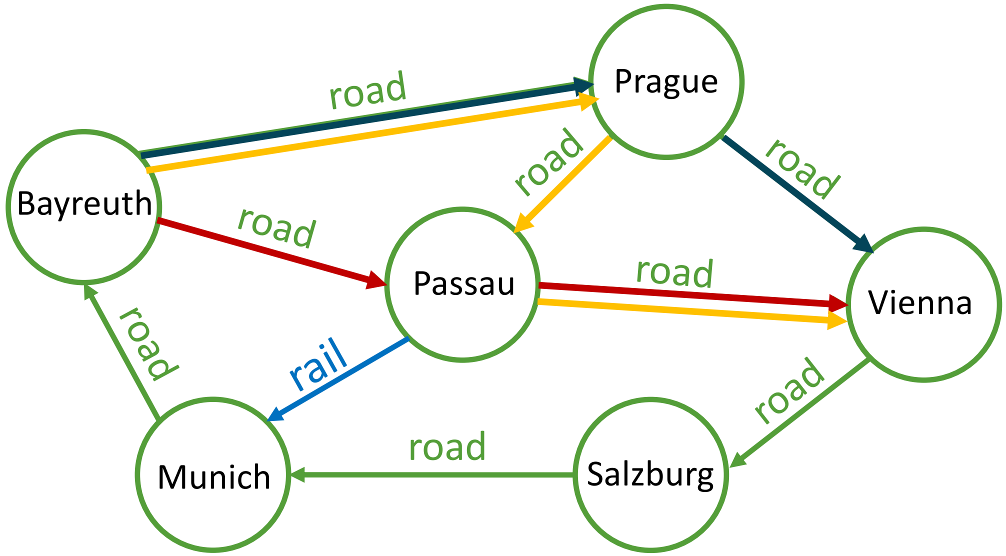

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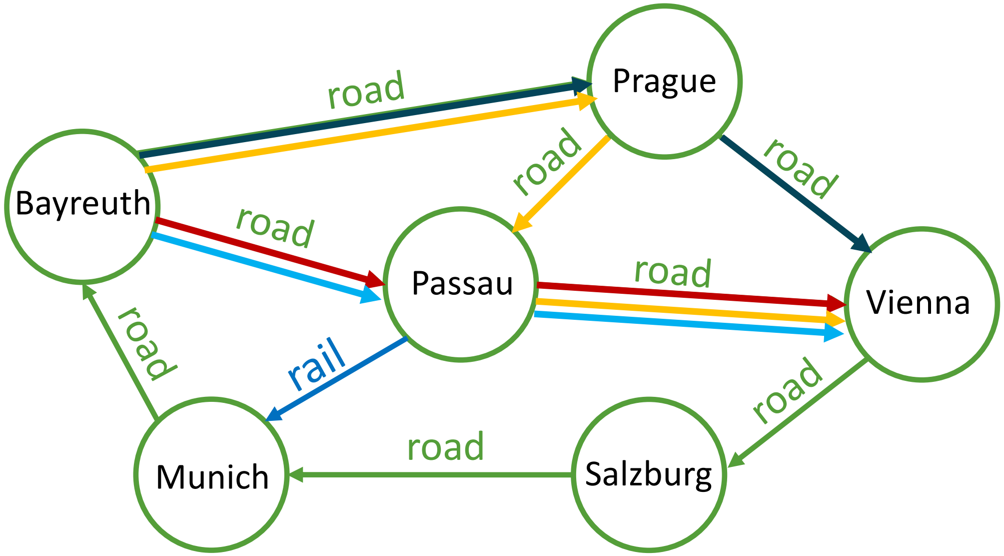

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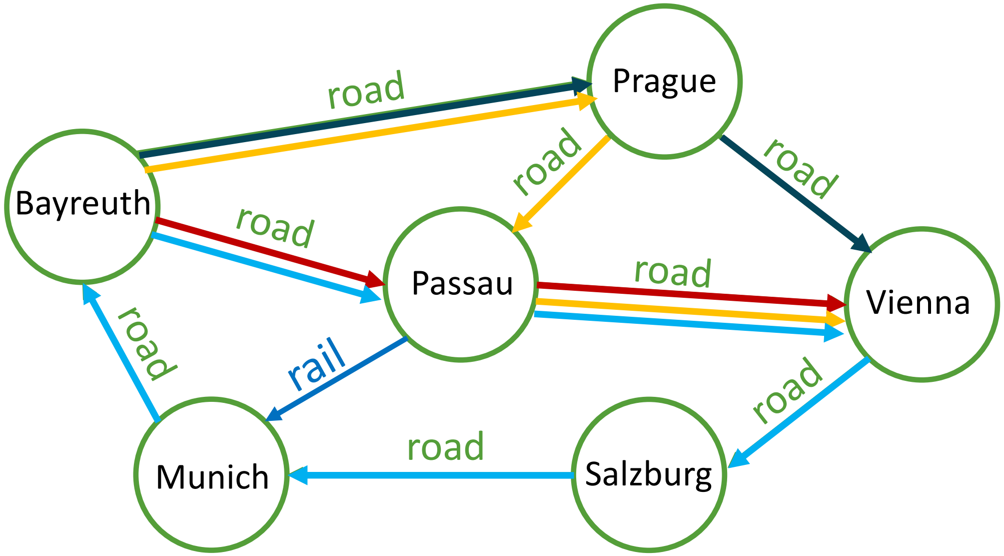

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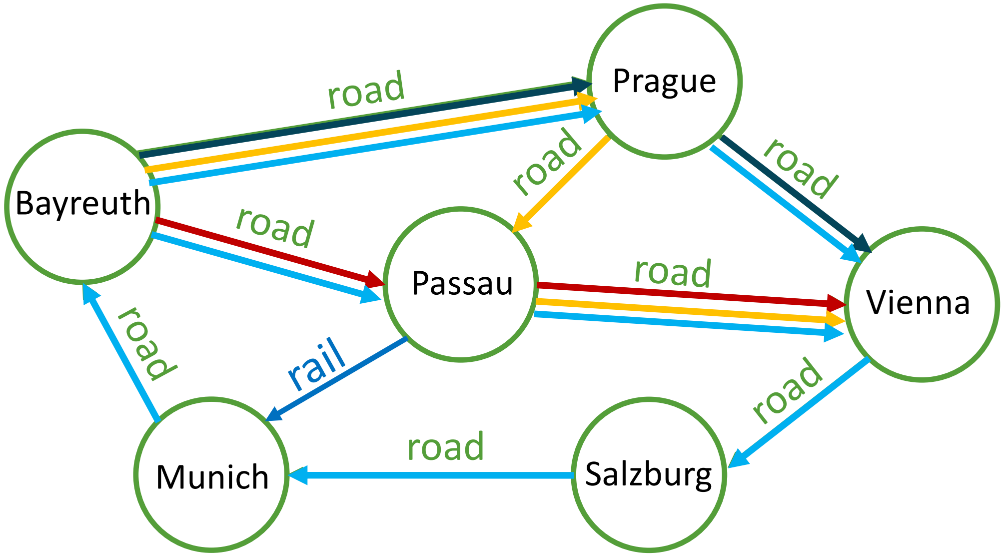

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The Point

There are different ways of matching paths in graphs

and any of them can make sense

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and any of them can make sense

But which variant do you want to use in a system?

Theory

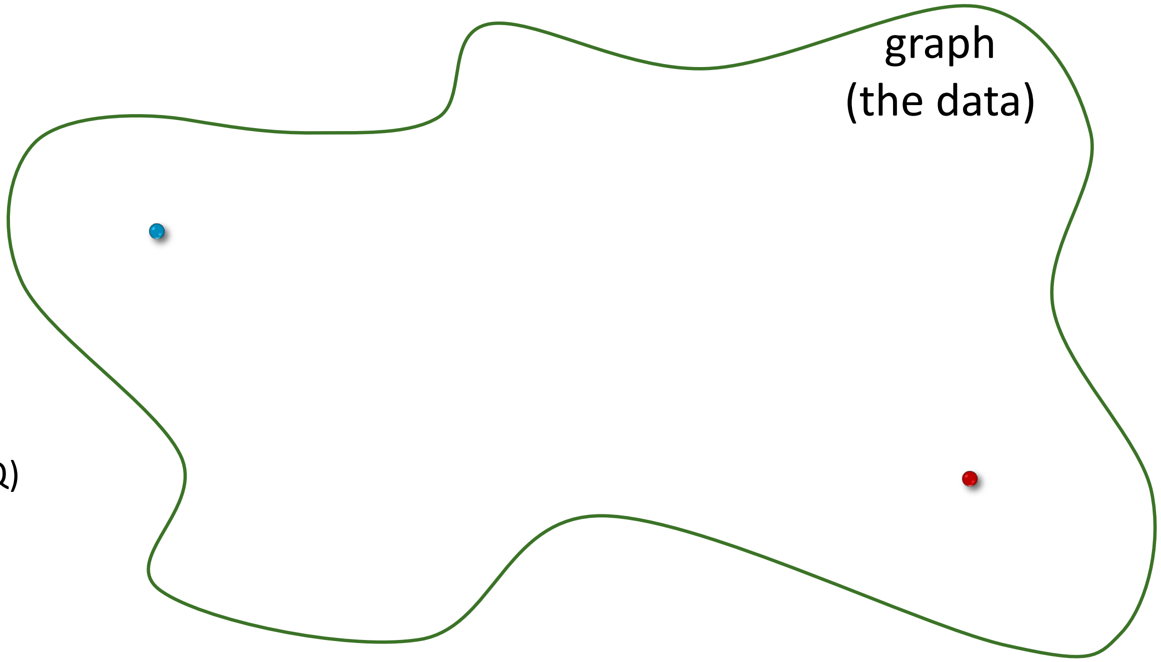
ON QUERYING PATHS IN GRAPH DATABASES

Computational Problems

Input

Regular expression r

called regular path query (RPQ)



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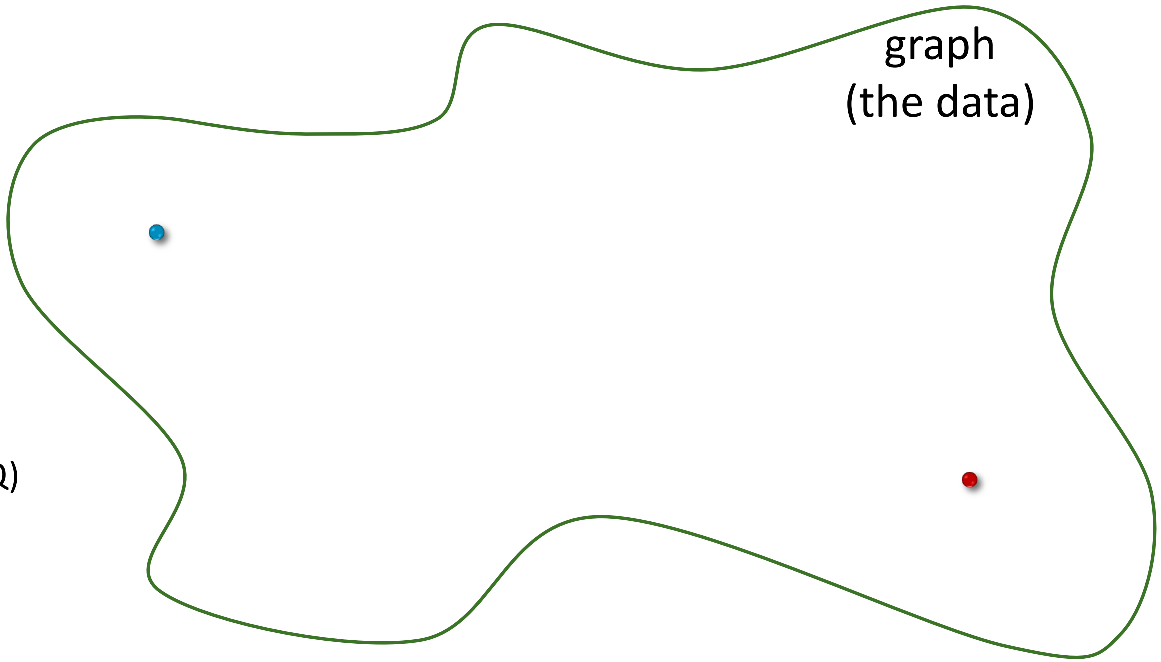

Regular expression r

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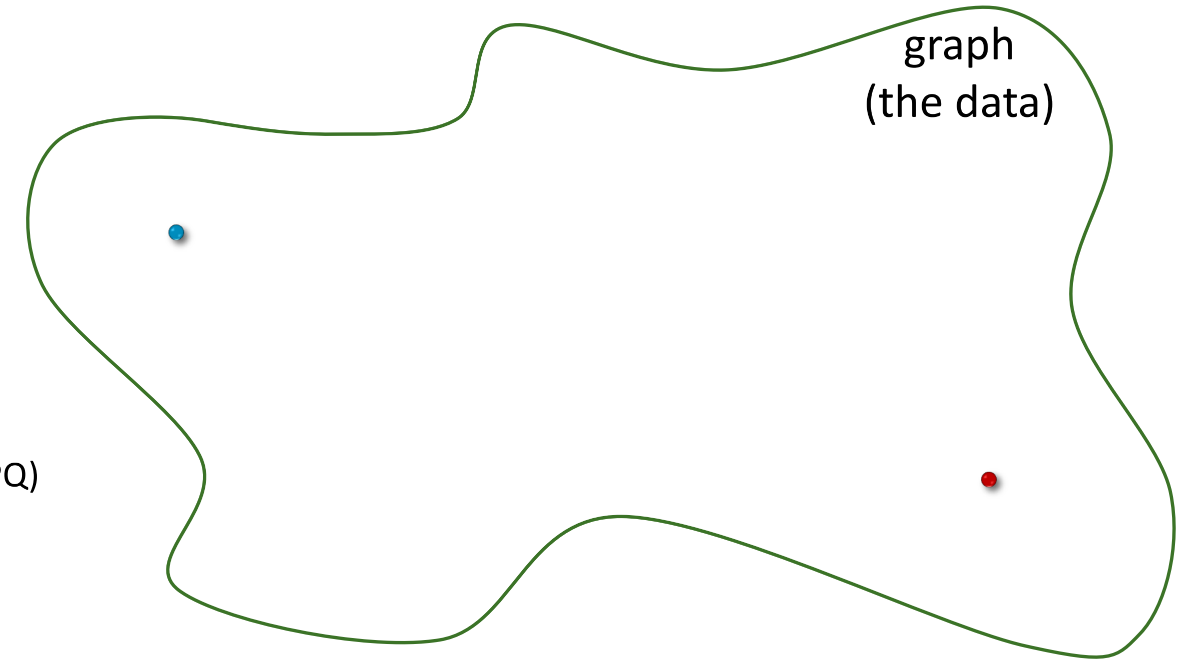
Problem

Path existence

Is there a path from [blue node] to [red node] that matches r?

Computational Problems

Input



Regular expression r

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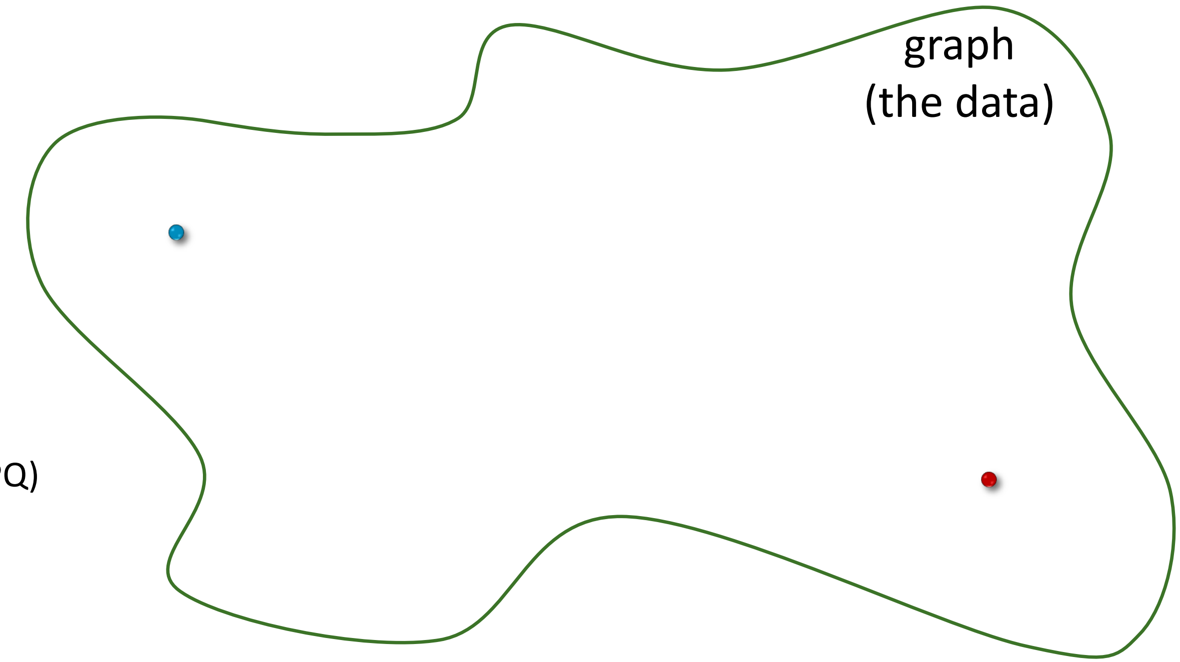
Problem

Path counting

How many paths from ● to ● match r?

Computational Problems

Input



Regular expression r
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Problem

Path enumeration

Enumerate the paths from • to • that match r

Considering Different Paths

Arbitrary paths

Boolean paths

Paths without node repetitions

Paths without edge repetitions

Considering Different Paths

Arbitrary paths

Boolean paths

Paths without node repetitions

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Considering Different Paths

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Arbitrary paths

Simple paths

Complexity of RPQ Evaluation

	Existence	Counting	Enumeration
Arbitrary paths			
Simple paths			

in P	**in FP**	**polynomial delay**
NP-hard	**#P-hard**	**too much delay**

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similar to
counting words in language of regular expression
#P-complete [Kannan et al., SODA 1995]

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Is there a simple path matching a^*ba^*?

NP-complete [Mendelzon, Wood, SICOMP 1995]

essentially because „simple path via a node" is NP-hard [Fortune et al., TCS 1980]

Is there a simple path matching $(aa)^*$?

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[Bagan, Bonifati, Groz PODS 2013]

Dichotomy for which expressions

the data complexity of this problem is **in P** or **NP-complete**

Theory VS Systems

Theory:

Systems:

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Theory: „Simple paths are computationally difficult, even for very small RPQs“

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Theory: „Simple paths are computationally difficult,
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Systems: „But we use simple paths and we‘re fine“

What is going on with these simple paths?

RPQs in SPARQL Query Logs

[Bonifati, M., Timm, PVLDB 2017]

Extracted 247,404 RPQs from SPARQL query logs (2009 - 2017)
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→ **Only very few different kinds of RPQs (± 17)**

RPQs in SPARQL Query Logs

Expression Type	Relative	Expression Type	Relative
A^*	48.76%	$a^*b?$	<0.01%
A	32.10%	abc^*	<0.01%
$a_1 \cdots a_k$	8.66%	$A_1 \cdots A_k$	<0.01%
a^*b	7.73%	$(ab^*) + c$	<0.01%
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$A?$	<0.01%		

$k \leq 6$

Disjunction of symbols: $A, A_1, \ldots$

Single symbols: $a, b, c, a_1, \ldots$

Data from [Bonifati et al., PVLDB 2017]

Simple Transitive Expressions

Atomic Expression

disjunction $(\boldsymbol{a_1} + \cdots + \boldsymbol{a_n})$ of symbols (denote this by $\boldsymbol{A}$, $\boldsymbol{A_i}$,...)

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„follow a path of length k"

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$L_1\ A^* L_2$ where L_1, L_2 are local expressions

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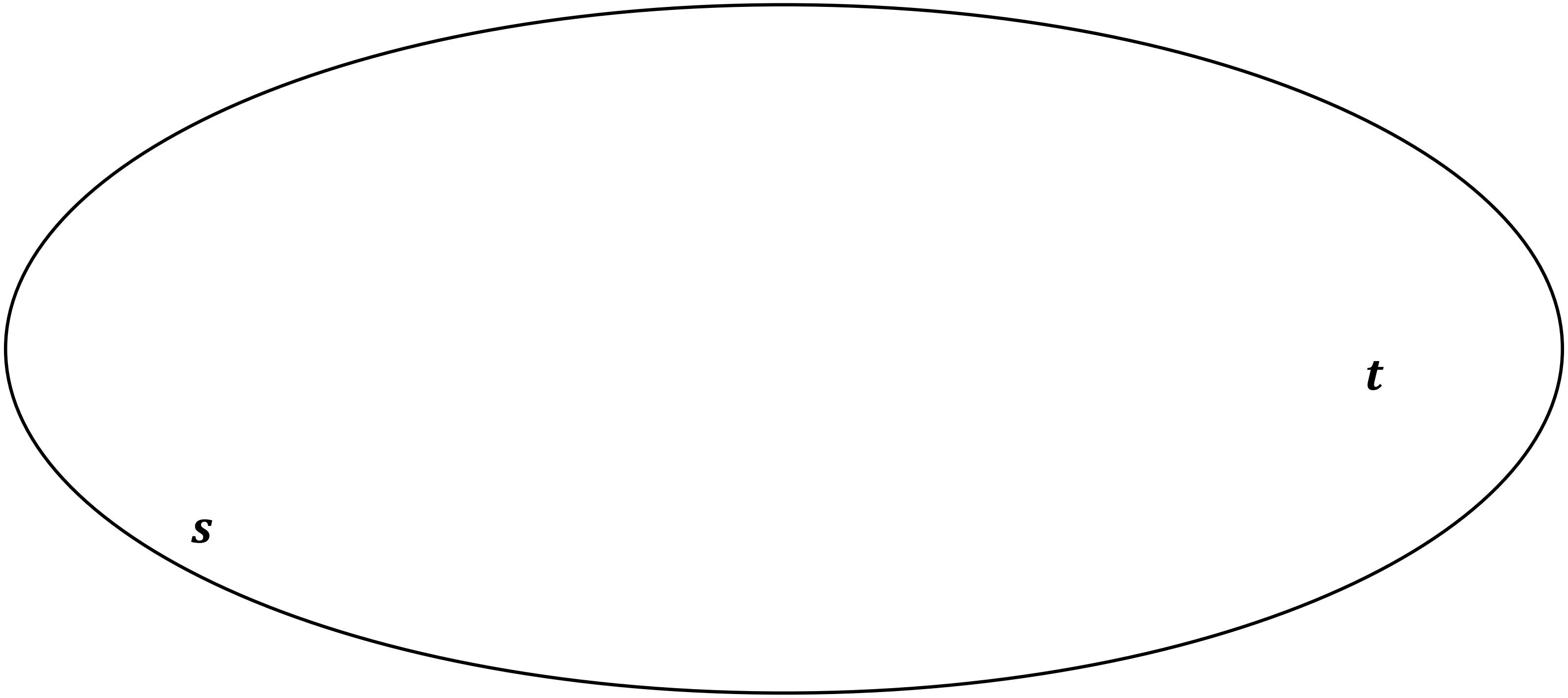

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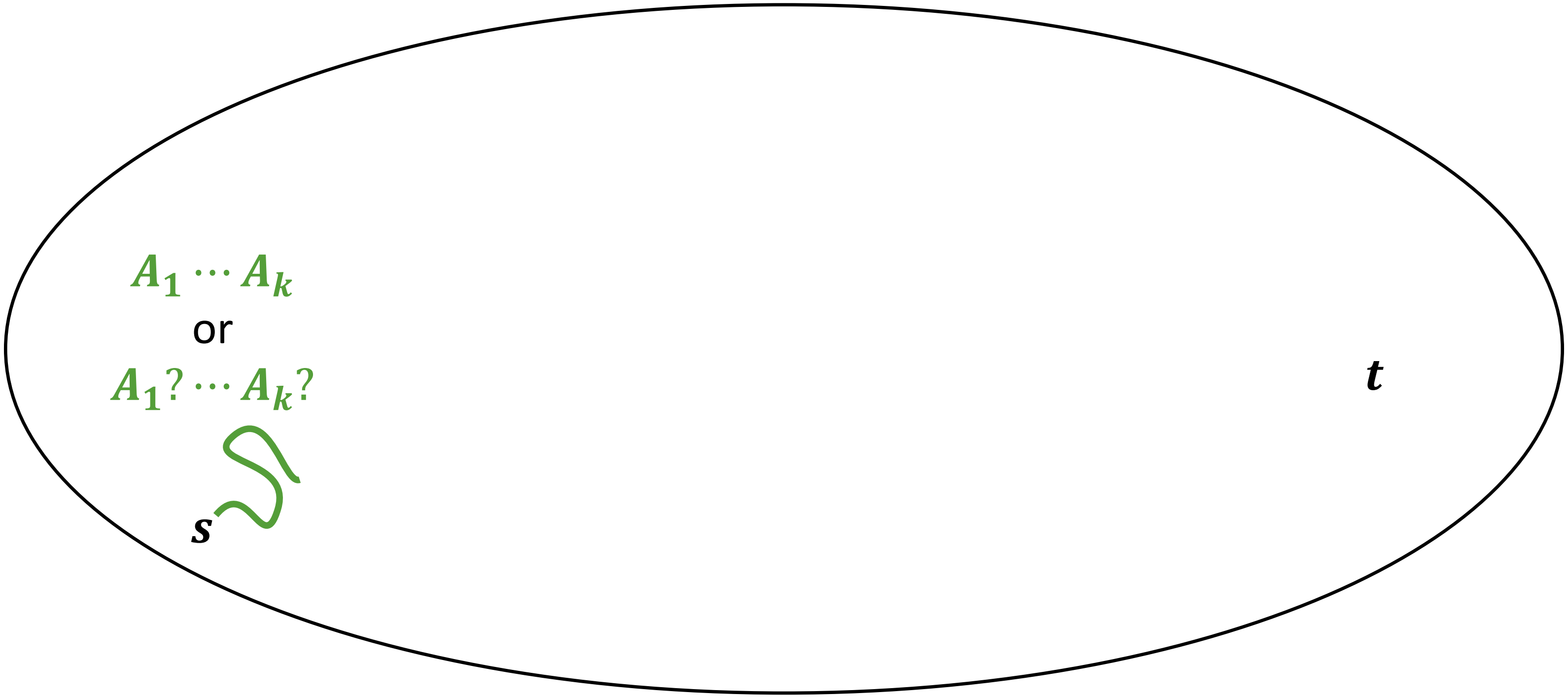

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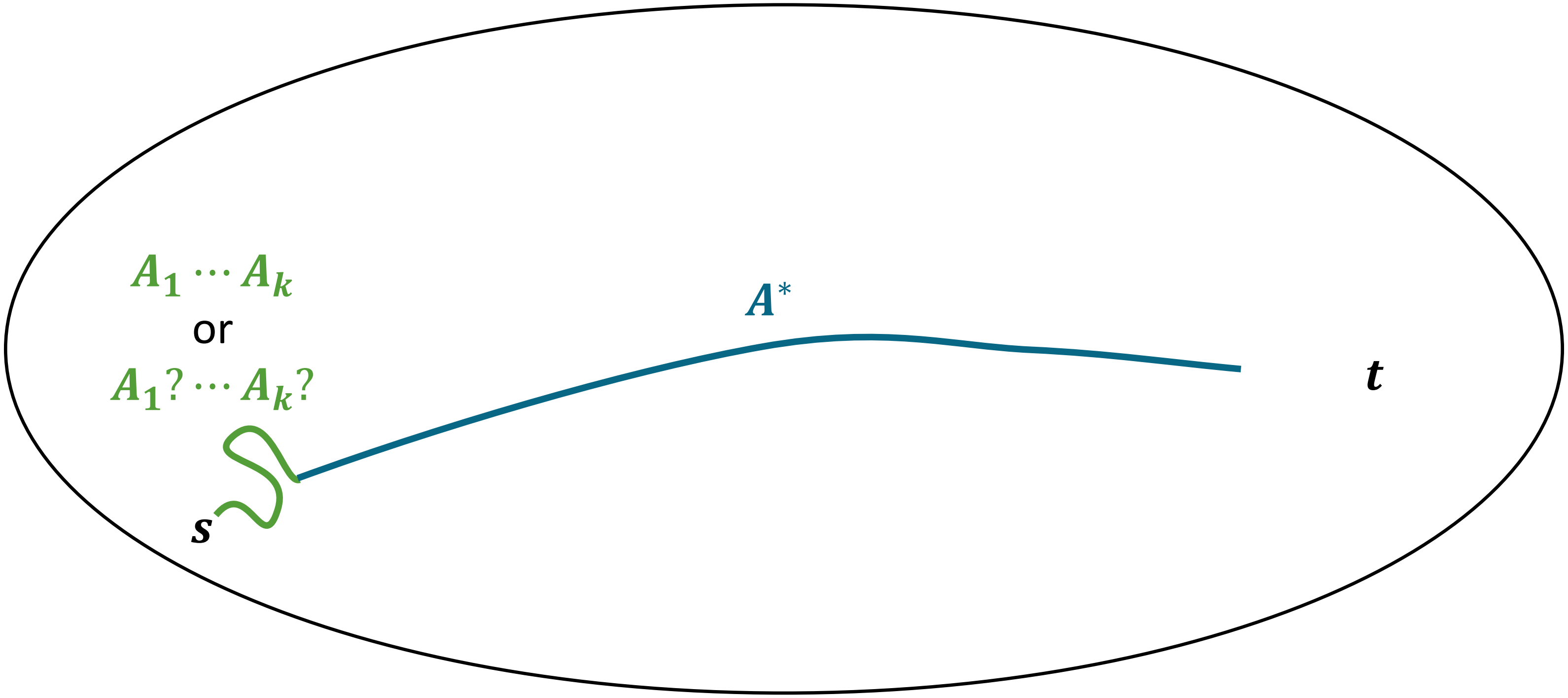

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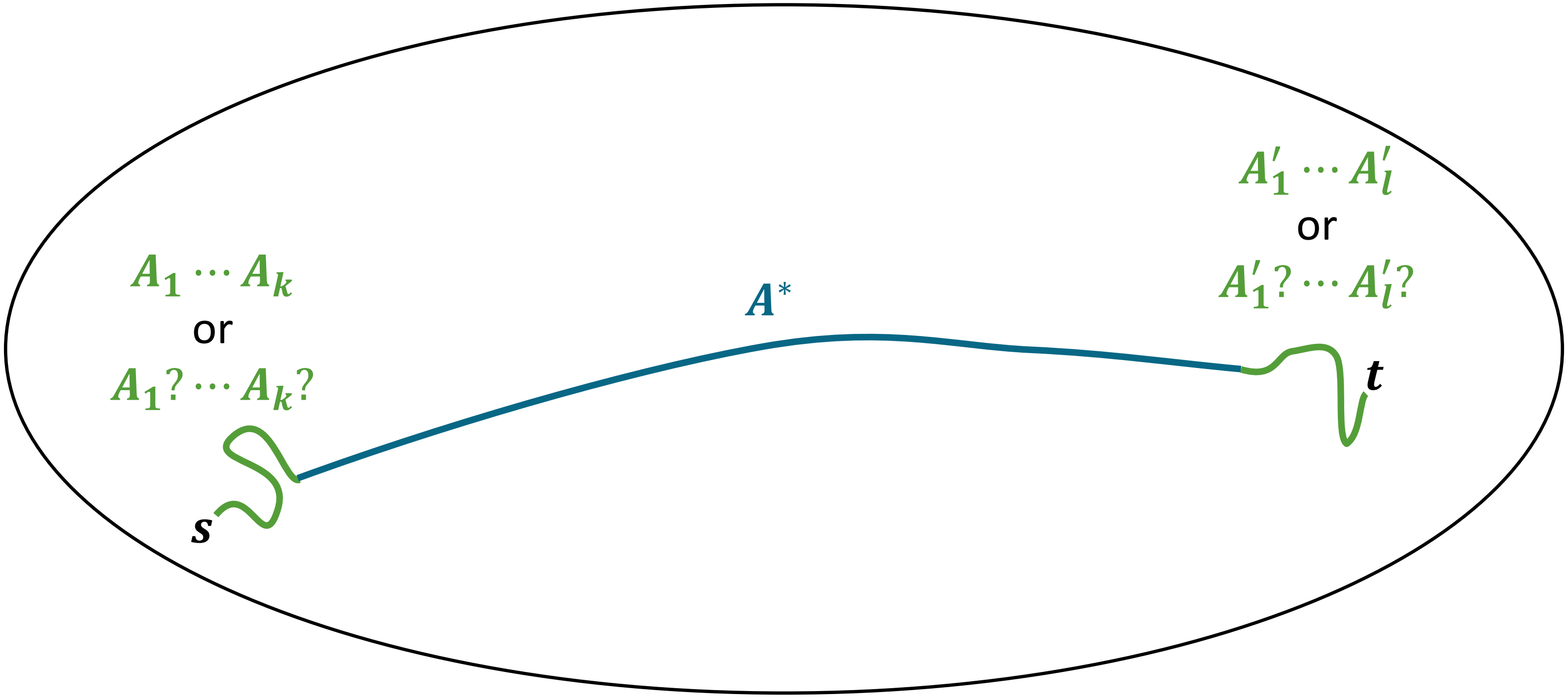

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STE

Union of STEs

something else

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$k \le$ [illegible]

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99.99% are STEs!

Data from [Bonifati et al., PVLDB 2017]

Main Theorem Warm-Up

Simple path existence

Given graph $\boldsymbol{G}$, nodes $\boldsymbol{s}$ and $\boldsymbol{t}$, and RPQ $\boldsymbol{r}$,
is there a simple path from $\boldsymbol{s}$ to $\boldsymbol{t}$ that matches $\boldsymbol{r}$?

Main Theorem Warm-Up

Simple path existence for $\boldsymbol{R}$

Given graph $\boldsymbol{G}$, nodes $\boldsymbol{s}$ and $\boldsymbol{t}$, and RPQ $\boldsymbol{r} \in \boldsymbol{R}$,
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Example classes $\boldsymbol{R}$:

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These are non-trivial problems!

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Color coding technique

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Theorem [Technique from Fomin et al., JACM 2016 communicated to us by Holger Dell]

"Simple path existence for $\{a^k a^* \mid k \in \mathbb{N}\}$ is in FPT"

Representative sets technique

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Main Theorem

Let $\boldsymbol{R}$ be a class(*) of STEs:
if $\boldsymbol{R}$ is ***cuttable***, then simple path existence for $\boldsymbol{R}$ is in **FPT**
otherwise, simple path existence for $\boldsymbol{R}$ is **W[1]-hard**.

(*) satisfying a mild condition, needed for W[1] hardness

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parameter: size of RPQ

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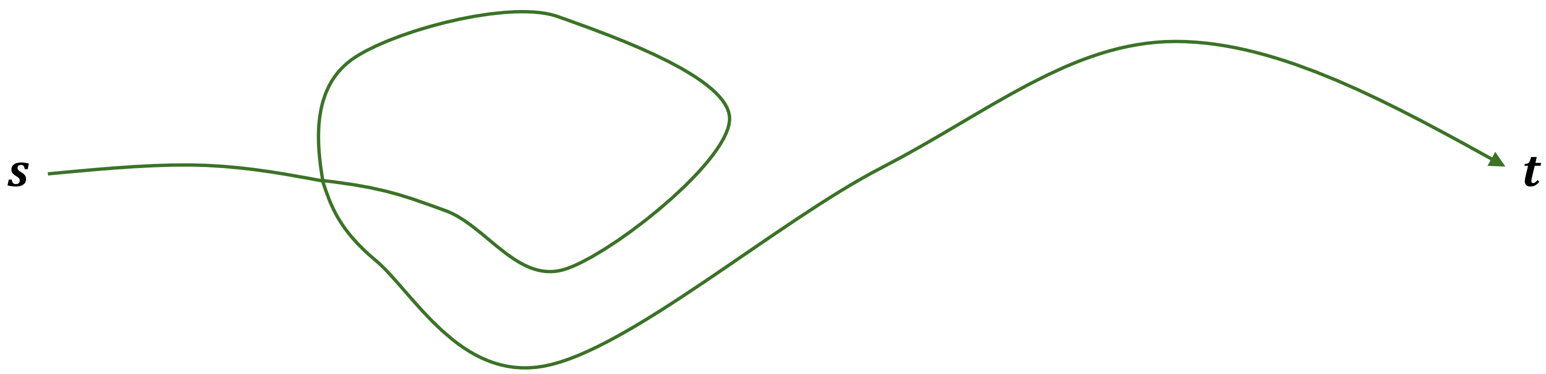
Intuition behind Cuttability



Path that matches $\boldsymbol{r}$

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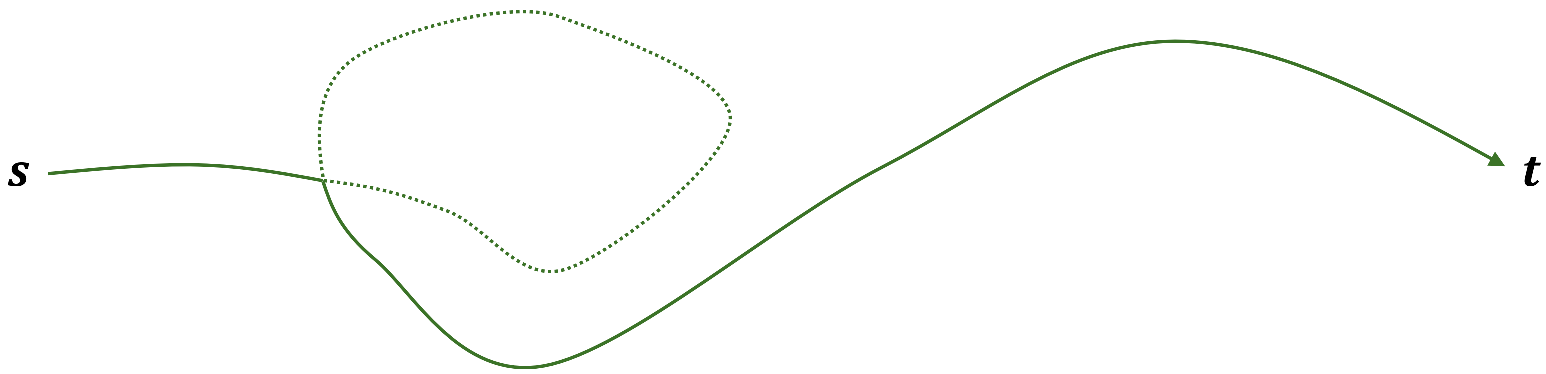

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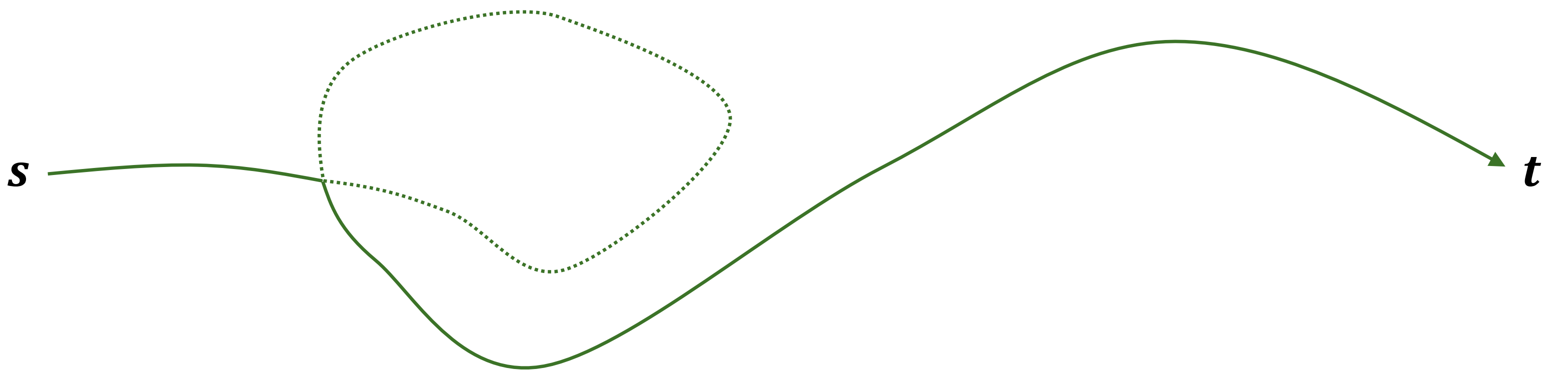
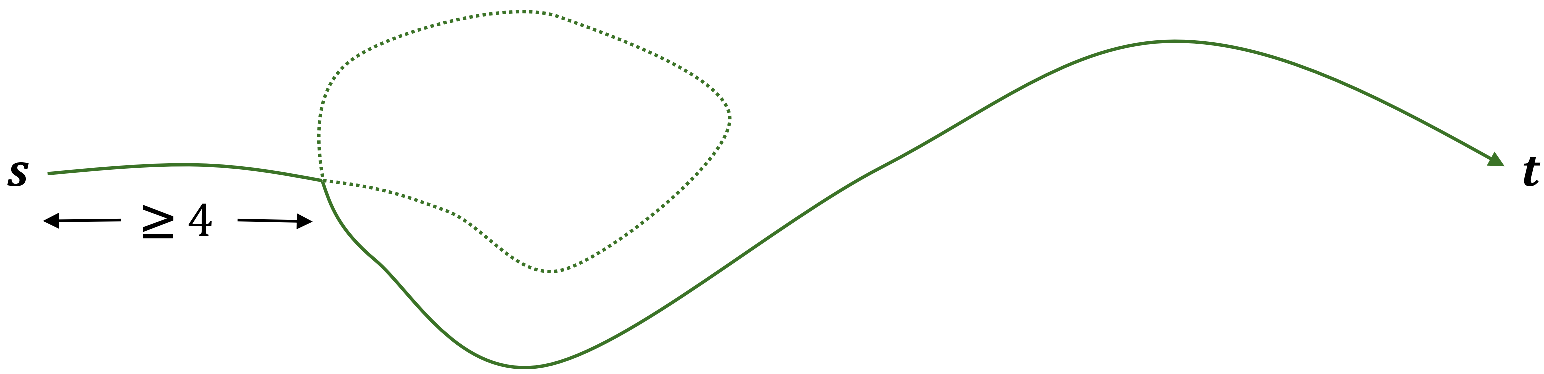


Simple Path that matches $\boldsymbol{r}$?

Does the simple path still match $\boldsymbol{r}$?

- "Easy" to check for $\boldsymbol{aaaaa}^*$ (check length)
- "Hard" to check for $\boldsymbol{bbbba}^*$ (check length+label)

Intuition behind Cuttability



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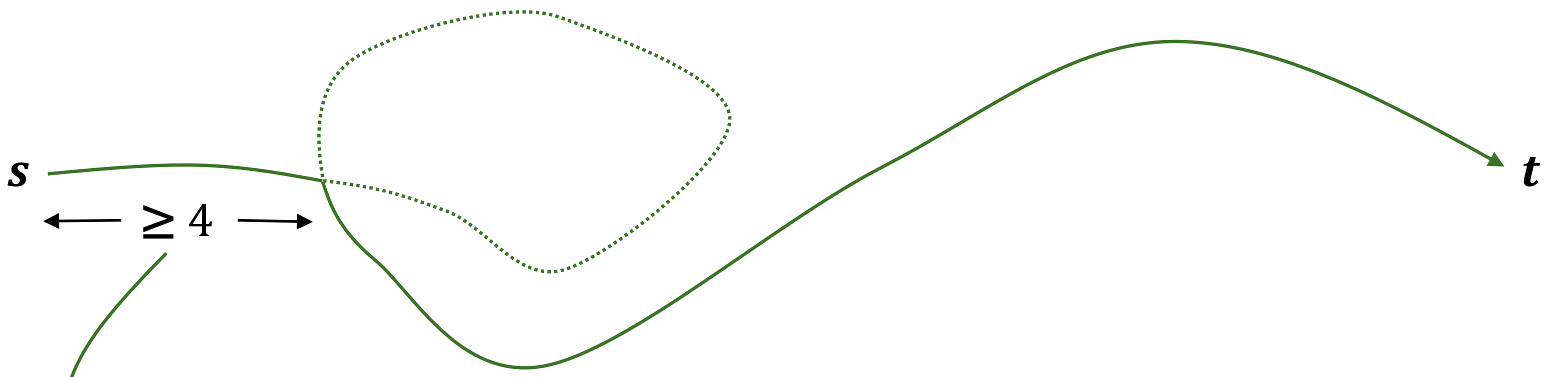

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cut border for $\boldsymbol{bbbba}^*$

Formalizing this Idea

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Consider STE $\boldsymbol{r} = \boldsymbol{A_1} \cdots \boldsymbol{A_k}\ \boldsymbol{A}^*$

Its **cut border** ℓ is the largest number such that $\boldsymbol{A} \not\subseteq \boldsymbol{A}_\ell$

(and $\ell = 0$ if no such $\boldsymbol{A}_\ell$ exists)

Formalizing this Idea

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Examples

- $\boldsymbol{aaaa^*}$ $\ell = 0$ because $\{\boldsymbol{a}\} \subseteq \{\boldsymbol{a}\}$
- $\boldsymbol{aaba^*}$ $\ell = 3$ because $\{\boldsymbol{a}\} \not\subseteq \{\boldsymbol{b}\}$
- $(\boldsymbol{a} + \boldsymbol{c})\boldsymbol{ab}(\boldsymbol{a} + \boldsymbol{b})^*$ $\ell = 3$ because $\{\boldsymbol{a}, \boldsymbol{b}\} \not\subseteq \{\boldsymbol{b}\}$

Formalizing this Idea

Consider STE $r = A_1 \cdots A_k \ A^*$

Its **cut border** ℓ is the largest number such that $A \not\subseteq A_\ell$
(and $\ell = 0$ if no such A_ℓ exists)

Examples

- $aaaa^*$ — $\ell = 0$ — because $\{a\} \subseteq \{a\}$
- $aaba^*$ — $\ell = 3$ — because $\{a\} \not\subseteq \{b\}$
- $(a+c)ab(a+b)^*$ — $\ell = 3$ — because $\{a, b\} \not\subseteq \{b\}$

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Definition

A class R of STEs is ***cuttable***, if
there is a constant c such that all its expressions have cut border $\leq c$

Formalizing this Idea

Definition

A class $\boldsymbol{R}$ of STEs is ***cuttable***, if
there is a constant $\boldsymbol{c}$ such that all its expressions have cut border $\leq \boldsymbol{c}$

parameter: size of RPQ

Main Theorem

Let $\boldsymbol{R}$ be a class$^{(*)}$ of STEs:
if $\boldsymbol{R}$ is ***cuttable***, then simple path existence for $\boldsymbol{R}$ is in **FPT**
otherwise, simple path existence for $\boldsymbol{R}$ is **W[1]-hard**.

For the **FPT** upper bound,
the complexity in the parameter is single exponential

Upper Bound Idea

Upper Bound Idea

Theorem [Alon, Yuster, Zwick, JACM 1995]

Finding **simple paths** of length **exactly k** is in FPT

Color coding technique

Theorem [Fomin et al., JACM 2016]

Finding **simple cycles** of length **at least k** is in FPT

Representative sets technique

Theorem [Technique from Fomin et al., JACM 2016 communicated to us by Holger Dell]

Finding **simple paths** of length **at least k** is in FPT

Representative sets technique

Upper Bound Idea

Find a simple path matching $\boldsymbol{A_1} \cdots \boldsymbol{A_k}\, \boldsymbol{A}^*$

$\boldsymbol{s}$ $\boldsymbol{t}$

Upper Bound Idea

Find a simple path matching $A_1 \cdots A_k \, A^*$

s $\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad$ t

$A_1 \cdots A_c$

Brute force

Upper Bound Idea

Find a simple path matching $\boldsymbol{A_1 \cdots A_k\, A^*}$

$\boldsymbol{s}$ $\boldsymbol{t}$

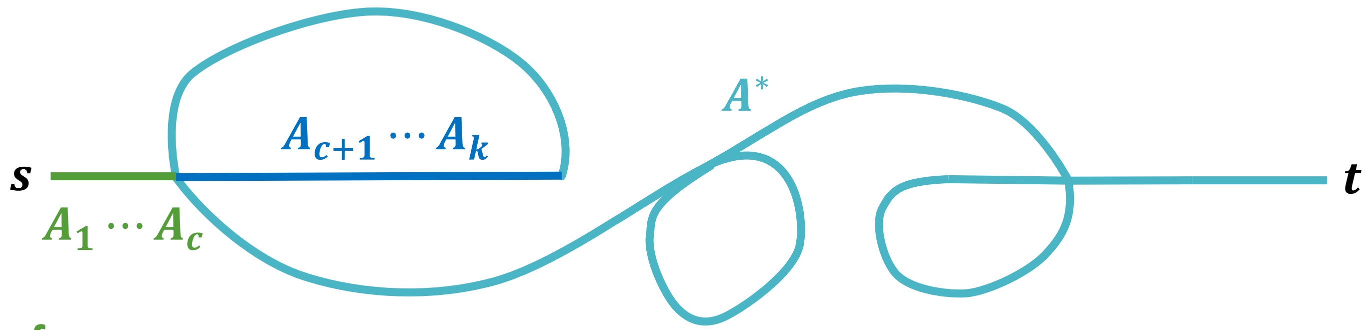
$\boldsymbol{A_1 \cdots A_c}$

Brute force

Simple Path matching $\boldsymbol{A_{c+1} \cdots A_k\, A^*}$
and avoiding the brute force part

Upper Bound Idea

Find a simple path matching $\boldsymbol{A_1 \cdots A_k\ A^*}$



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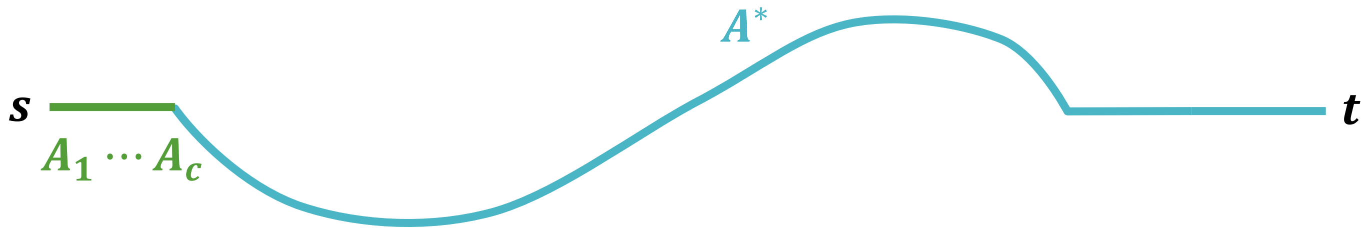

Upper Bound Idea

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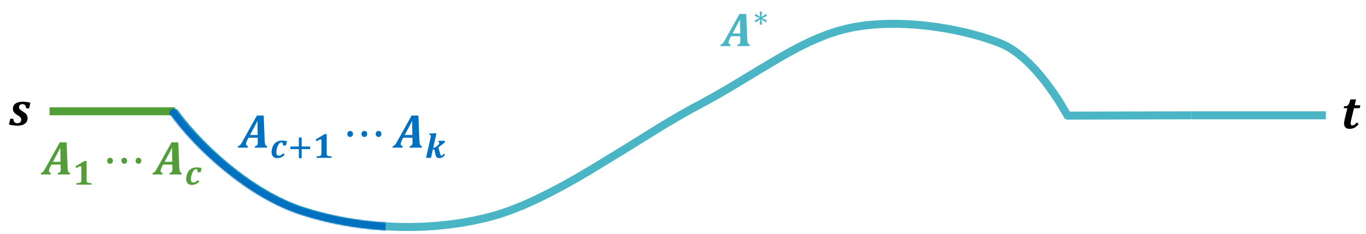


Since $A \subseteq A_i$

Lower Bound Idea

Parameterized Two Disjoint Paths

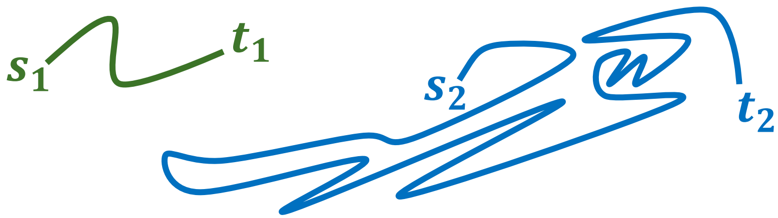
s_1 t_1 s_2 t_2

Lower Bound Idea

Parameterized Two Disjoint Paths

Given graph G,
nodes s_1 and t_1 and s_2 and t_2
and a parameter **k**

Are there node-disjoint paths
from s_1 to t_1
from s_2 to t_2



Lower Bound Idea

Parameterized Two Disjoint Paths

Given graph G,

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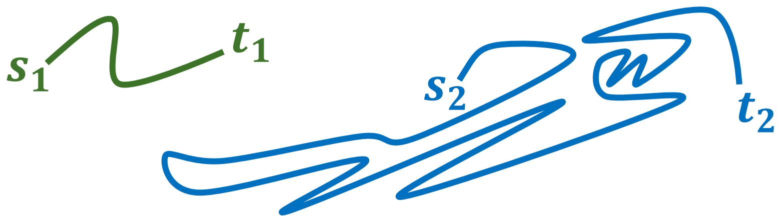
Are there node-disjoint paths

from s_1 to t_1 of length at most **k**

from s_2 to t_2



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Theorem (Main Technical Result)

Parameterized Two Disjoint Paths is **W[1]-hard**

Building on proofs from [Slivkins, SIDMA 10; Grohe&Grüber ICALP 07]

Lower Bound Idea

Theorem (Main Technical Result)

Parameterized Two Disjoint Paths is **W[1]-hard**

Lemma

Let $\boldsymbol{R}$ be a class(*) of STEs:

if $\boldsymbol{R}$ is ***not cuttable***, then simple path existence for $\boldsymbol{R}$ is **W[1]-hard**.

Lower Bound Idea

Theorem (Main Technical Result)

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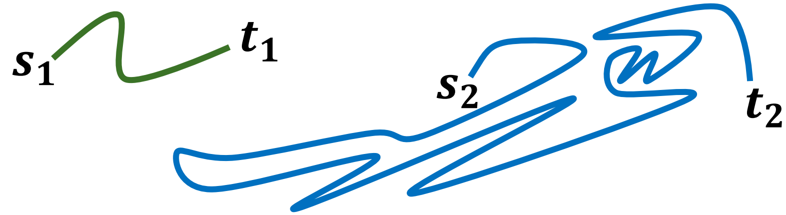

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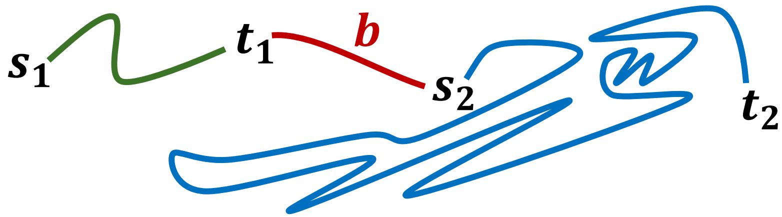

Warning: drastic oversimplification

Lemma

Let $\boldsymbol{R}$ be a class(*) of STEs:

if $\boldsymbol{R}$ is ***not cuttable***, then simple path existence for $\boldsymbol{R}$ is **W[1]-hard**.

Extensions

Extensions

Main Theorem

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The main result extends to:

- Enumeration problems
  FPT time becomes FPT delay — using [Yen 1971]
- Edge-disjoint paths
  But the dichotomy slightly changes — [ArXiv 2017]

Taking a Step Back

WHAT DID WE LEARN HERE?

So what does all this mean?

Expression Type	Relative	Expression Type	Relative
A^*	48.76%	$a^*b?$	<0.01%
A	32.10%	abc^*	<0.01%
$a_1 \cdots a_k$	8.66%	$A_1 \cdots A_k$	<0.01%
a^*b	7.73%	$(ab^*) + c$	<0.01%
A^+	1.54%	$a^* + b$	<0.01%
$a_1? \cdots a_k?$	1.15%	$a + b^+$	<0.01%
$aA?$	0.01%	$a^+ + b^+$	<0.01%
$a_1 a_2? \cdots a_k?$	0.01%	$(ab)^*$	<0.01%
$A?$	<0.01%		

$k \leq 6$

Cuttable STEs ($\ell \leq 2$) Thus in FPT

Union of STEs

something else

- These expressions have **cut border** $\leq$ **2**
- The FPT algorithms have parameter $\boldsymbol{k} \leq$ **6**

- **But even naive algorithms are expected to work reasonably well (brute-force checks for paths of lengh 2 and simple paths of length 6)**

Take Home Messages

- Looking in query logs can pay off and inspire new research questions!
- 99.99% of RPQs found in a practical study are
  Simple Transitive Expressions (STEs)
- Dichotomy for simple path evaluation of STEs
  - Another one for no-repeated-edge semantics is similar
- If “cut borders are bounded”, evaluation of STEs is FPT
  - Cut borders in the real data are at most 2
  - “FPT parameters” in the real data are 6 (for exact length) and 2 (for minimum length)

Thank you!