Typechecking Top-Down XML Transformations

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Overview

- Introduction
- Schema : Tree Languages
- Tree Transformations : XSLT
- The Typechecking Problem
- Main Results
- Proof Ideas
- Conclusion and Future Work

Importance of Typechecking

An example:

Suppose that a certain user community agrees to produce documents satisfying a common tree type τ.

For a user, who executes an XML to XML transformation T, an input tree type τ_{in} is available.

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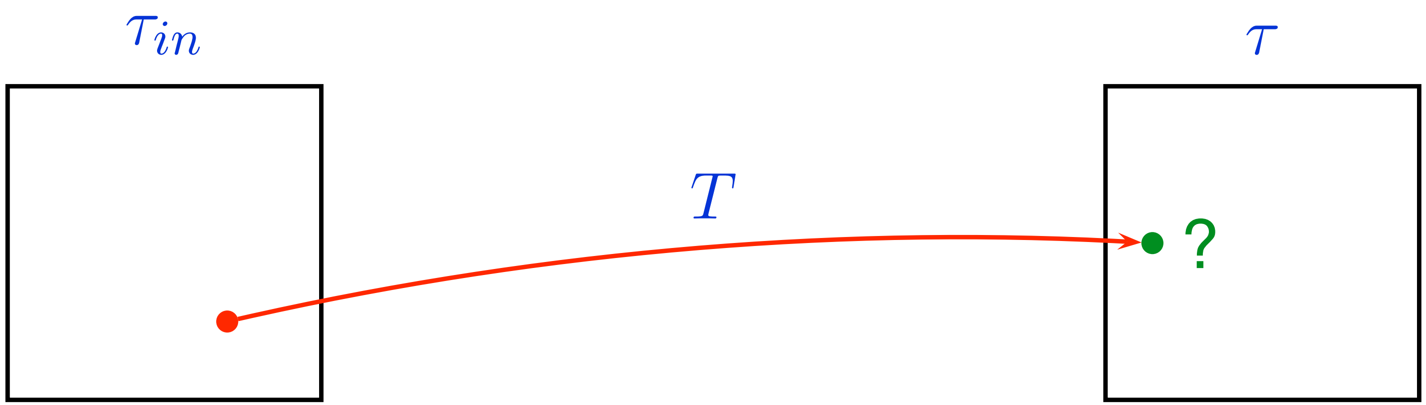

This is called the *Typechecking Problem*.

XML

```
<food>
    <dish type="Belgian">
        <potato> fries </potato>
        <sauce> mayonnaise </sauce>
    </dish>
    <dish type="Italian">
        <pasta> spaghetti </pasta>
        <vegetable> tomato </vegetable>
    </dish>
</food>
```

XML

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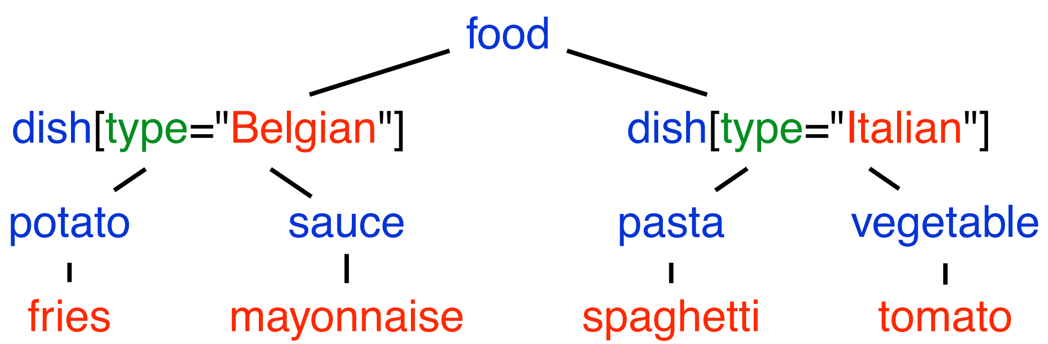

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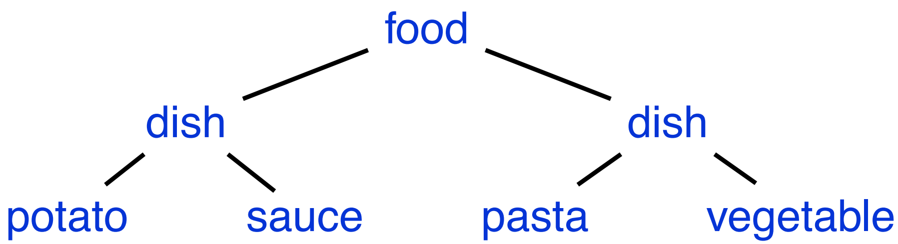

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  - Complexity is high (non-elementary).

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  - Complexity is high (non-elementary).

We try to lower complexity by simplifying the schema languages and the tree transformations.

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Tree languages

- DTDs:

  food → (dish)*

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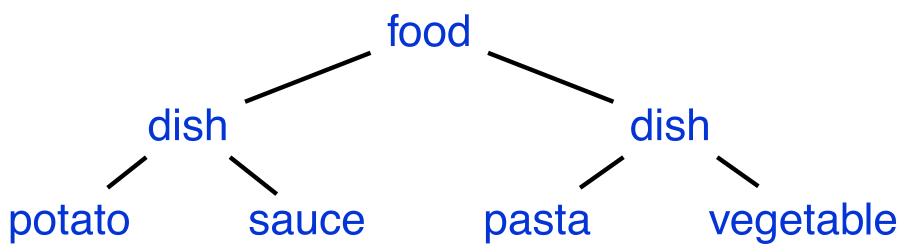

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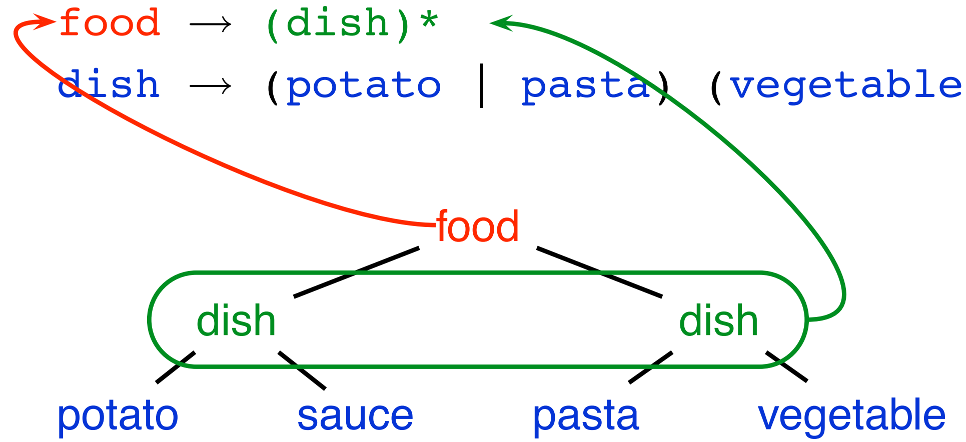

Tree languages

- DTDs:

  $\texttt{food} \longrightarrow (\texttt{dish})^*$

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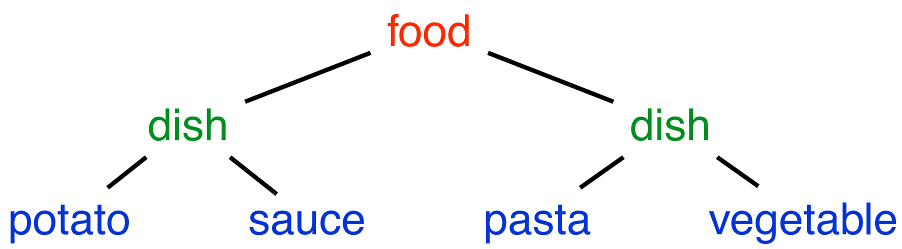

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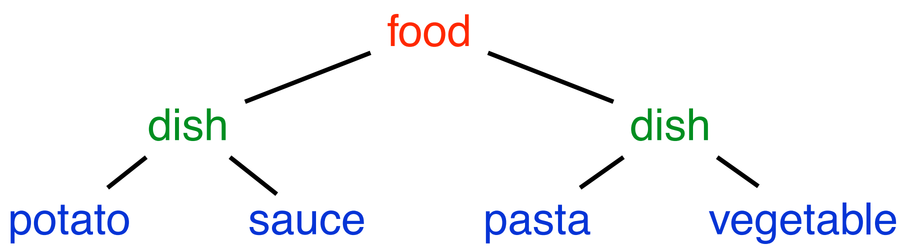


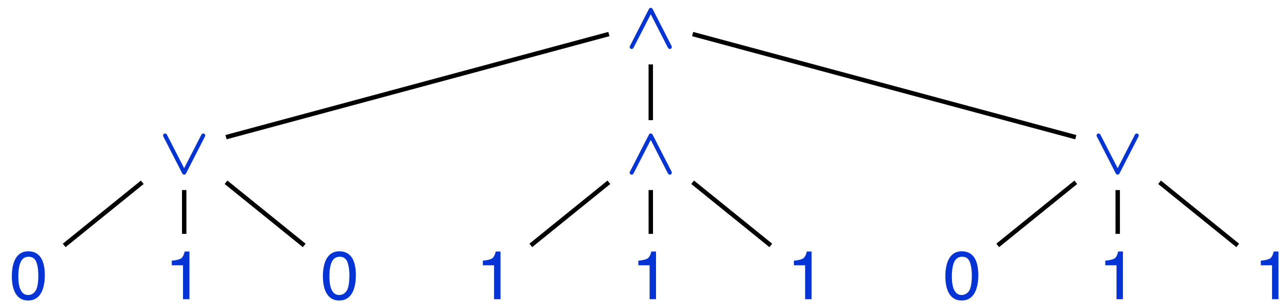
- Tree Automata:

  unranked tree automata

  Specify transition function δ by regular string languages over states.

Tree Automata - Example

Evaluate Boolean expressions:



States: $\{t, f\}$

	label		state		language
$\delta($	1	,	t	$) =$	ε
$\delta($	0	,	f	$) =$	ε
$\delta($	$\wedge$	,	t	$) =$	tt^*
$\delta($	$\wedge$	,	f	$) =$	$(f\|t)^*f(f\|t)^*$
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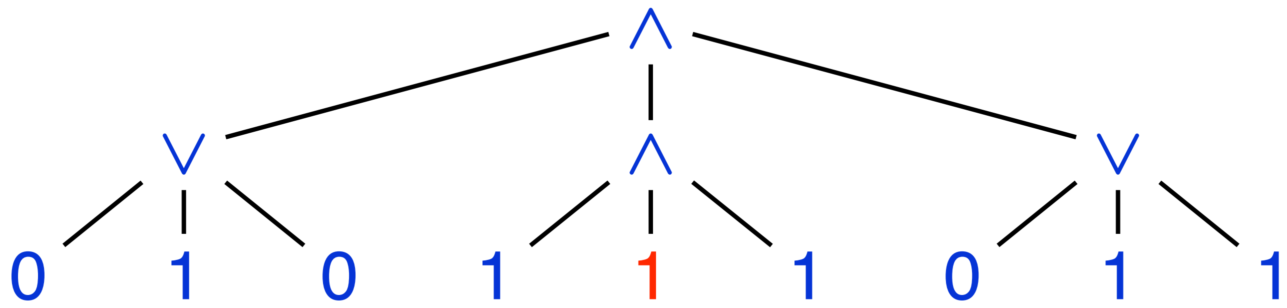

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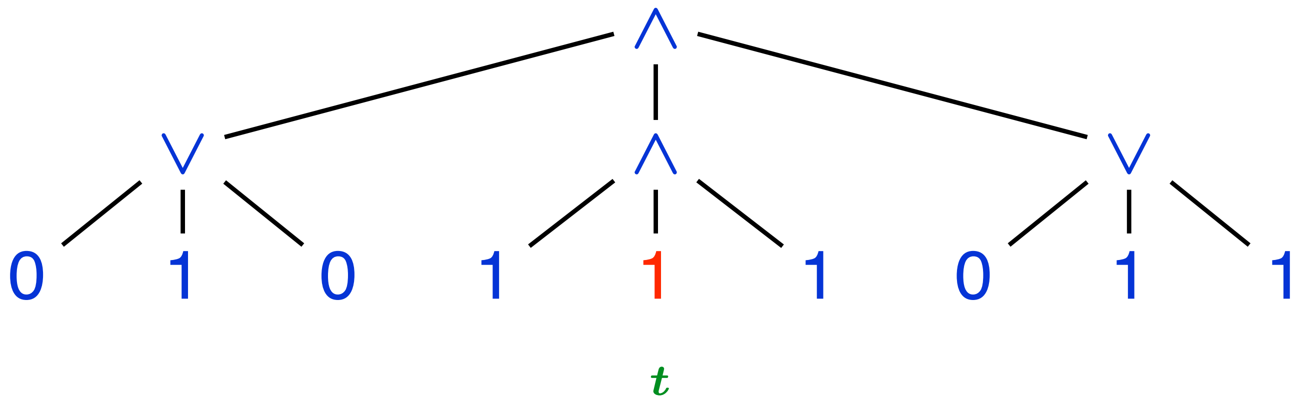

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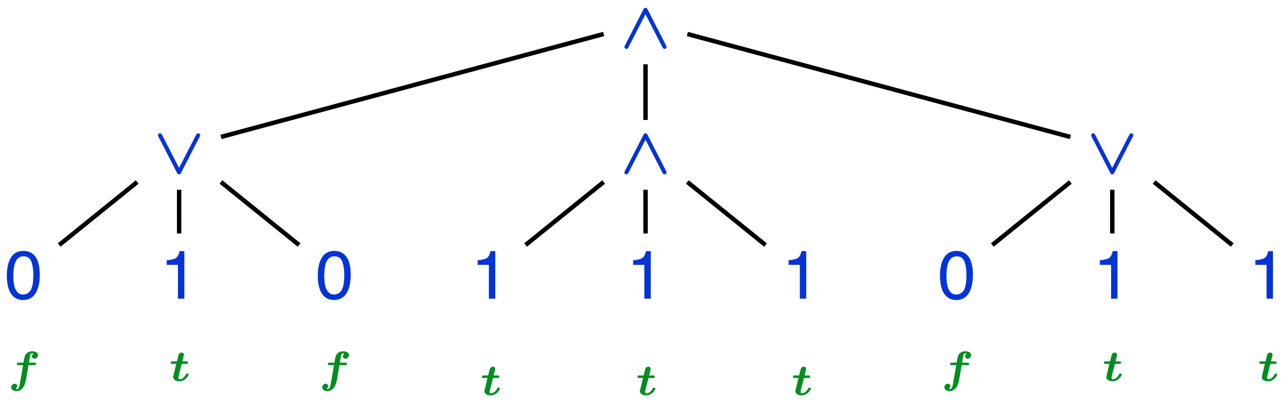

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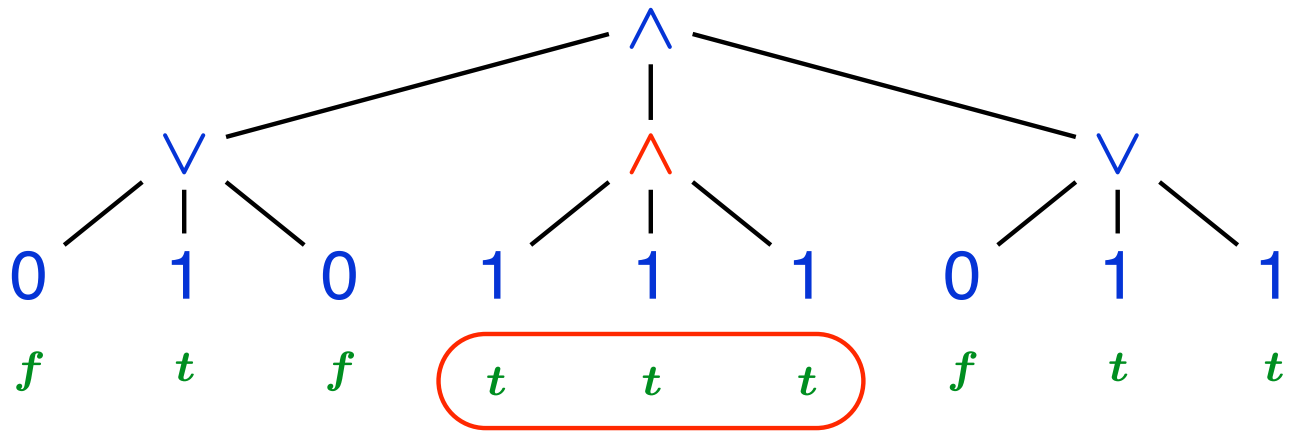

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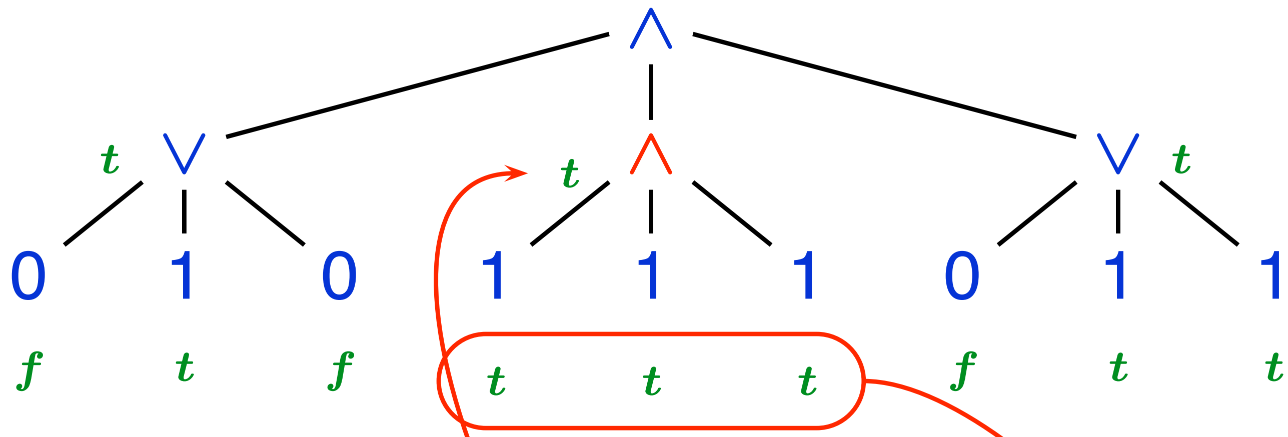

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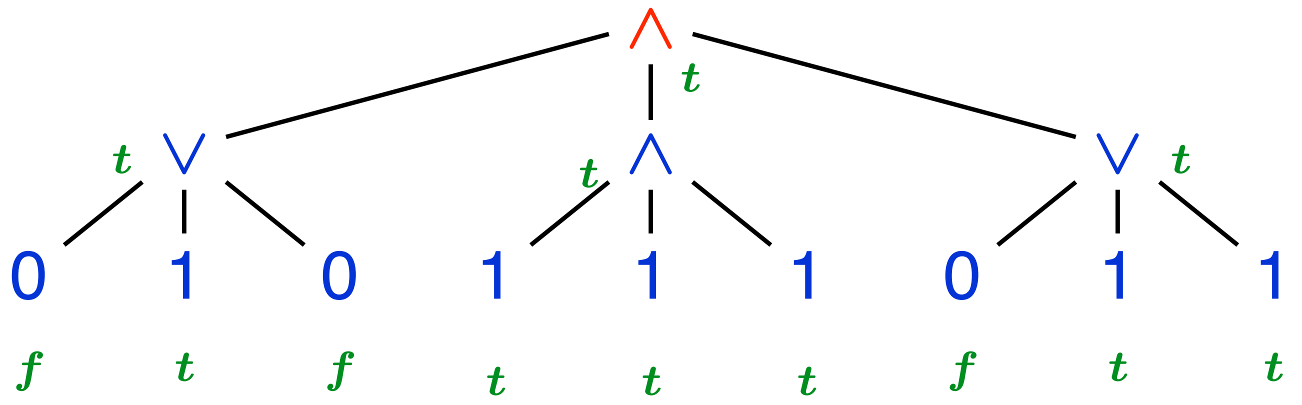

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XSLT - simple case

Example : 1 mode : simple

$(a, \text{simple}) \longrightarrow \begin{array}{c} b \\ | \\ \text{simple} \end{array}$

$(b, \text{simple}) \longrightarrow \begin{array}{c} a \\ | \\ \text{simple} \end{array}$

XSLT - simple case

Example : 1 mode : simple

$(a,\text{simple}) \rightarrow$ b
|
simple

$(b,\text{simple}) \rightarrow$ a
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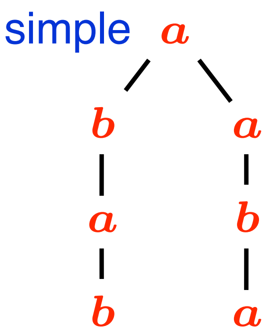
simple



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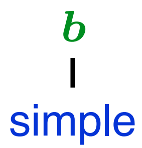

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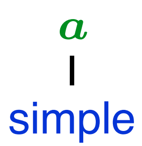

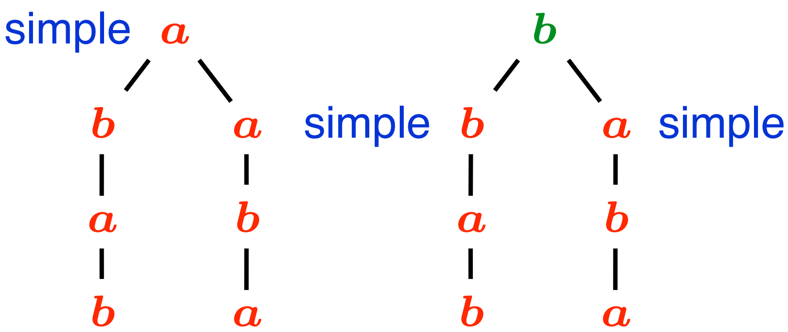

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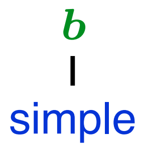

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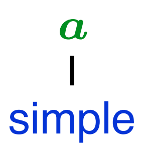

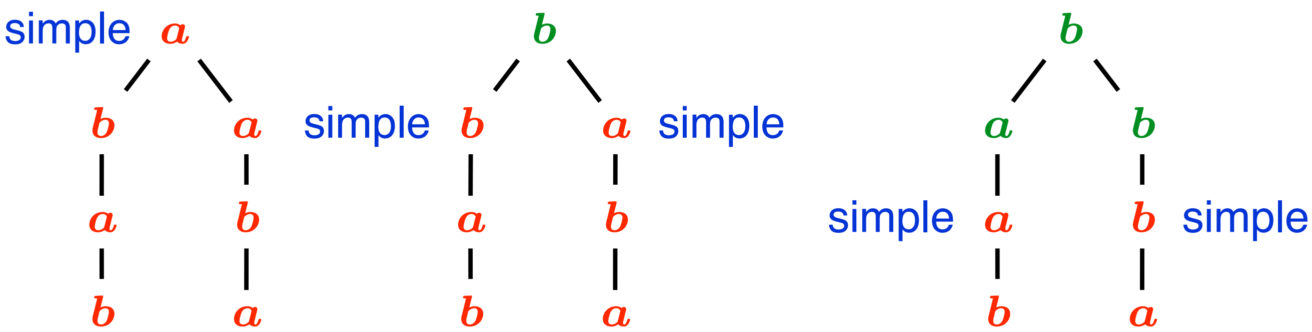

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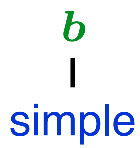

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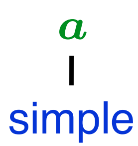
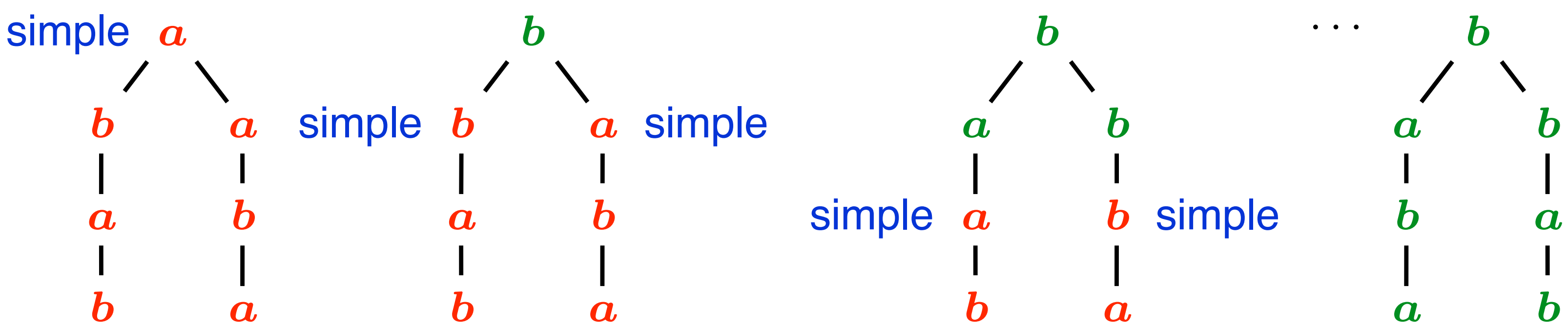
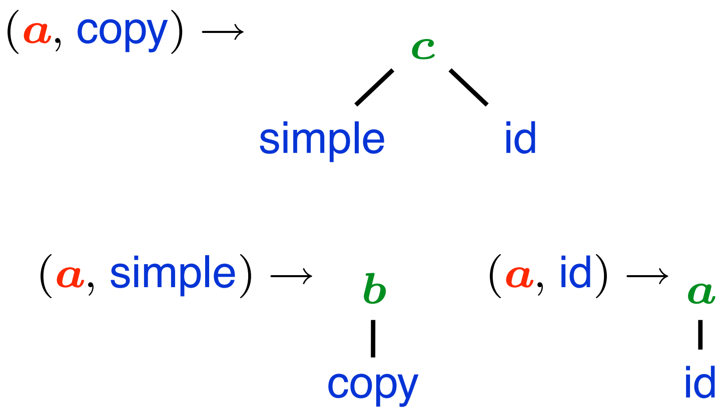




XSLT - copying

Example: 3 modes: simple, copy, id



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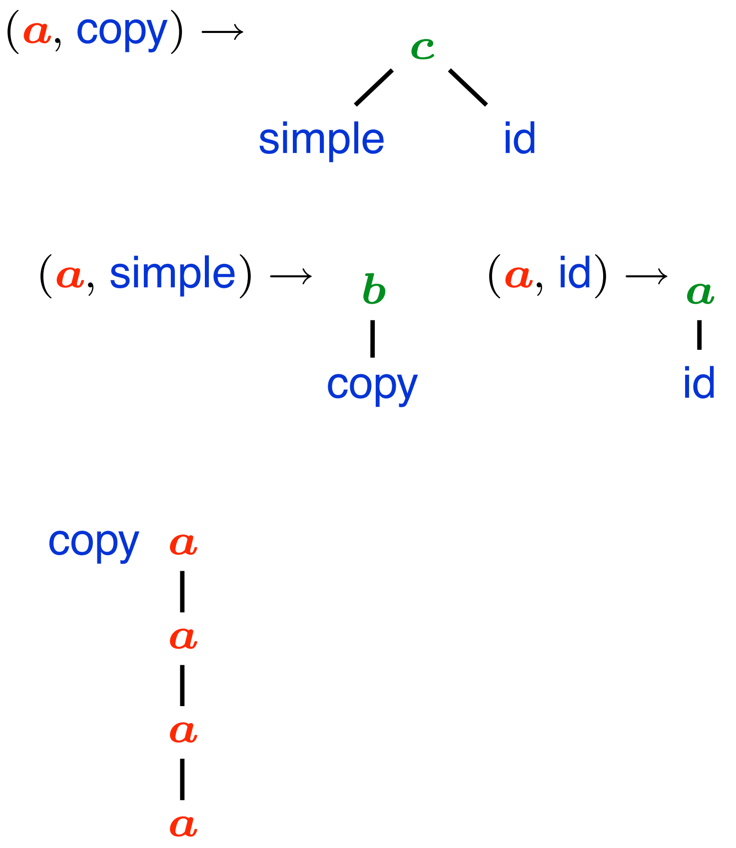

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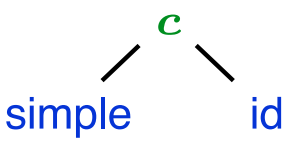

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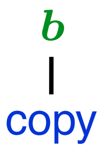

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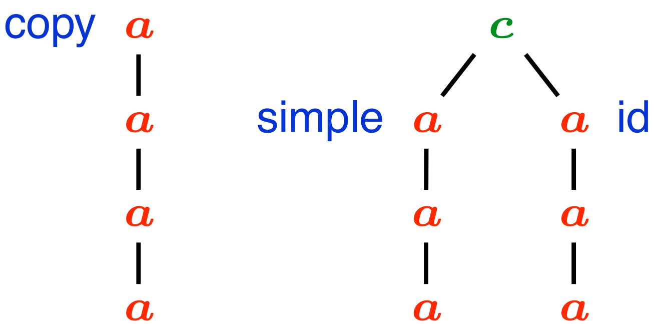

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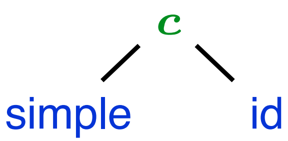

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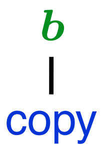

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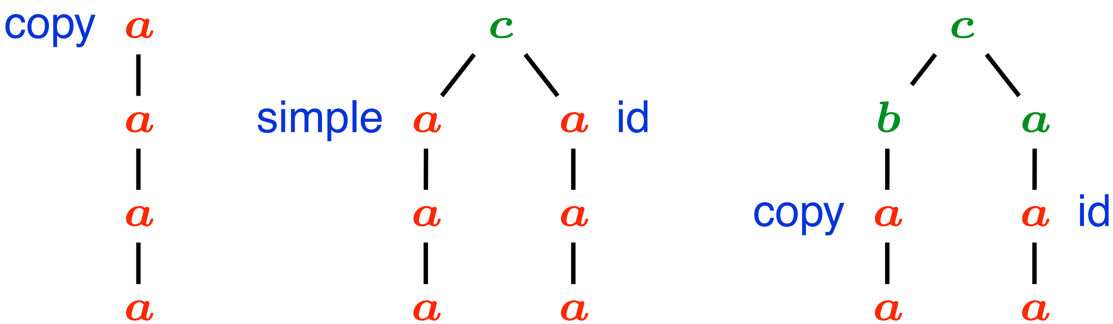

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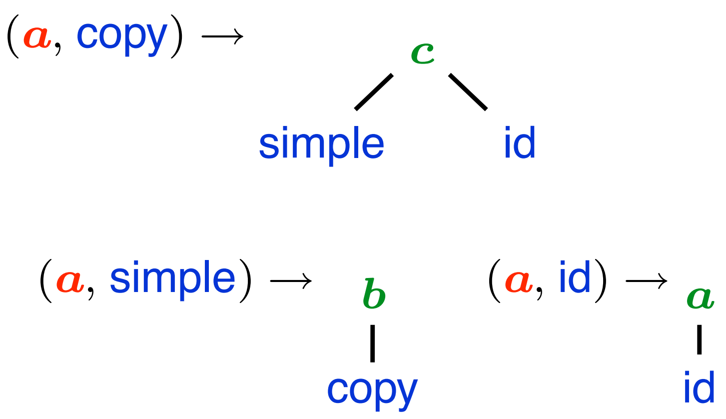

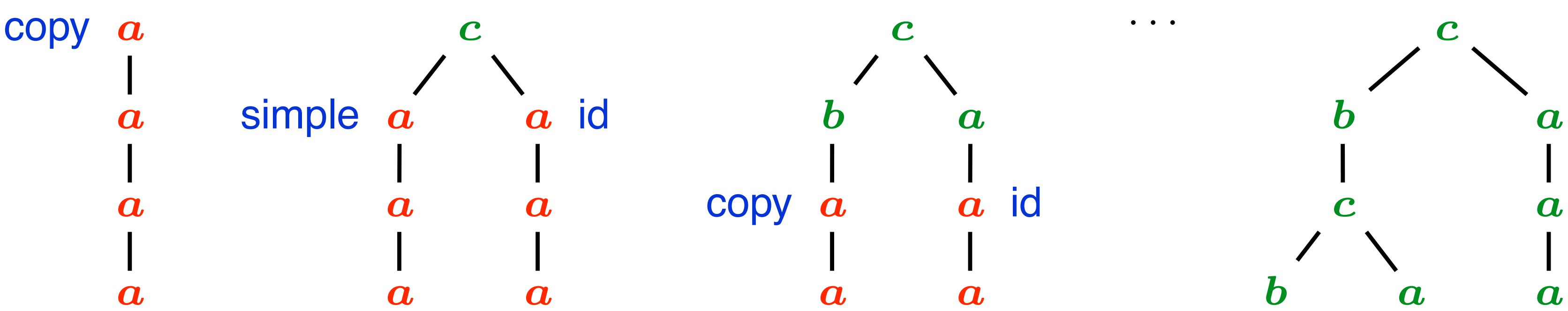

XSLT - deleting

Example: 2 modes: delete, copy

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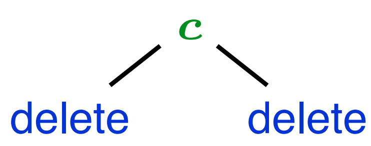

$(a, \text{delete}) \rightarrow \text{copy}$

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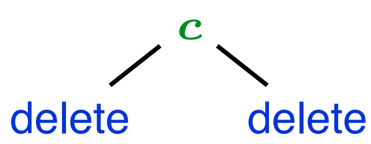
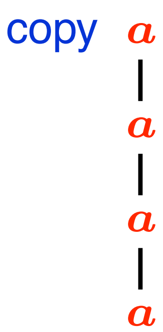
Example: 2 modes: delete, copy

$(a,$ copy$) \to$



$(a,$ delete $) \to$ copy



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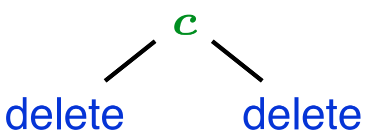

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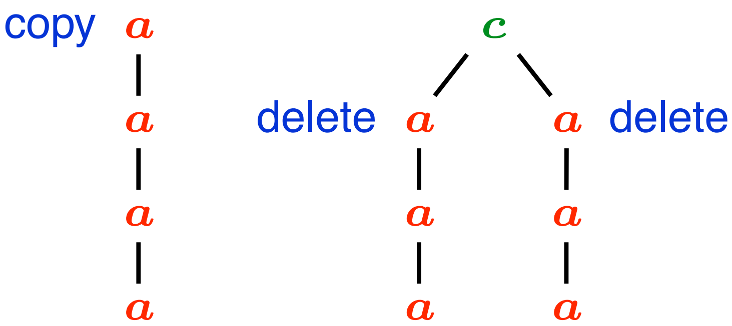

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$(a, \text{copy}) \rightarrow c(\text{delete}, \text{delete})$

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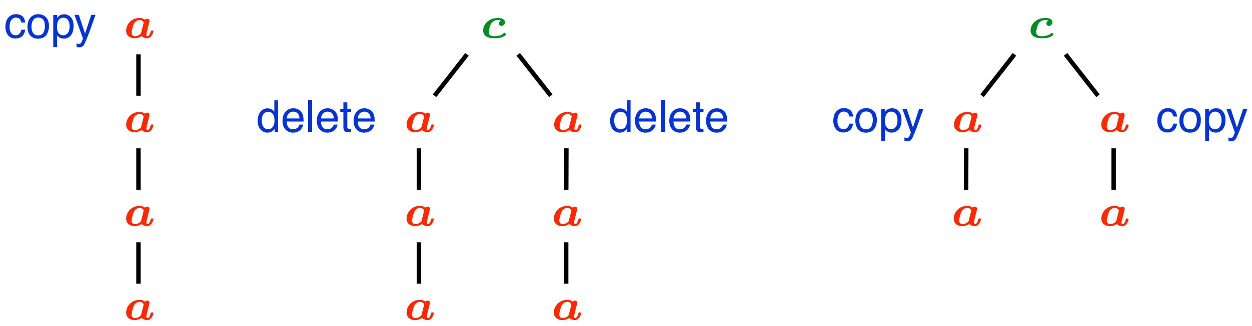

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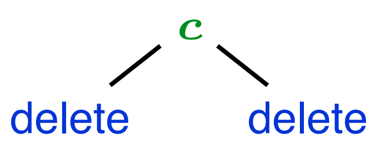

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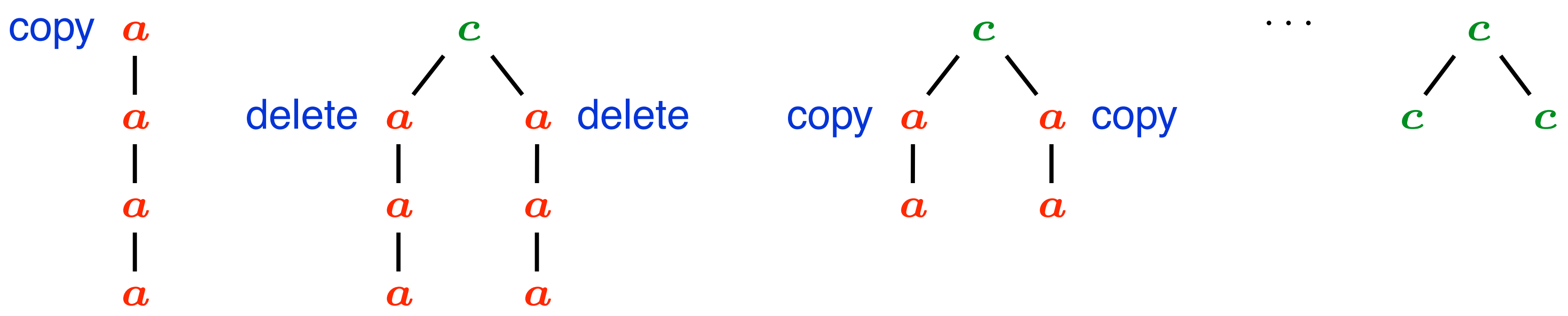

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Given:

- input tree language τ_{in} DTD, TA
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Is it true that,

$$\forall t \in \tau_{in} \Rightarrow T(t) \in \tau_{out}?$$

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Typechecking is complete for the complexity classes shown here.

Tree Automata: Toolbox

We can use different formalisms to represent the regular languages in tree automata.

Then, we can use the tree automata as a *toolbox*:

1. Emptiness of TA(2AFA) is in **PSPACE**.
2. Emptiness of TA(NFA) is in **PTIME**.

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Copying, DTD(RE) $\rightarrow$ in PSPACE

Reduction to emptiness of TA(2AFA).

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Reduction to emptiness of TA(2AFA).

Construct a TA(2AFA) B such that

$$L(B) = \{t \in \tau_{in} \mid T(t) \notin \tau_{out}\}.$$

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Reduction to emptiness of TA(2AFA).

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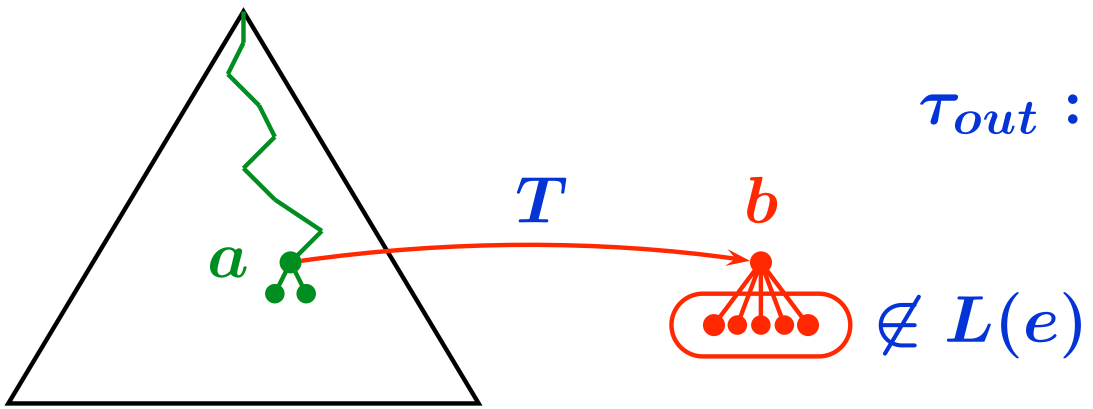

Main Results

What is the complexity of the typechecking problem?

	Tree Automata(NFA)	DTD(RE)	DTD(DFA)
deleting + copying	EXPTIME	EXPTIME	EXPTIME
no deleting + copying	EXPTIME	PSPACE	PSPACE
no deleting + bounded copying	EXPTIME	PSPACE	PTIME

Bound. Copying, DTD(DFA) $\rightarrow$ in PTIME

Look at previous reduction to emptiness TA(2AFA).

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Now:

- transformations can make only a bounded (fixed) number of copies
- DFAs are used

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Look at previous reduction to emptiness TA(2AFA).

Now:

- transformations can make only a bounded (fixed) number of copies
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So...

- 2-way no longer needed
- alternation no longer needed

And emptiness of TA(NFA) $\in$ **PTIME**.

Overview

- Introduction
- Schema : Tree Languages
- Tree Transformations : XSLT
- The Typechecking Problem
- Main Results
- Proof Ideas
- Conclusion and Future Work

Conclusion and Future Work

If we eliminate

- unbounded copying and deleting in the tree transformation
- non-determinism in the schema languages

the typechecking problem becomes **PTIME**-complete.

In the future, we will try to expand the **PTIME**-result.