Enumeration on Trees with Tractable Combined Complexity and Efficient Updates

Antoine Amarilli[1], Pierre Bourhis[2], Stefan Mengel[3], **Matthias Niewerth**[4]

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Dramatis Personae

Antoine Amarilli

Pierre Bourhis

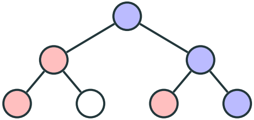
Stefan Mengel

Matthias Niewerth

Problem statement

MSO query evaluation on trees

Data: a tree T where nodes have a color from an alphabet 



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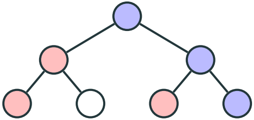

Query Q: a **formula** in monadic second-order logic (MSO)

- $P_{\bigcirc}(x)$ means "x is blue"
- $x \rightarrow y$ means "x is the parent of y"

"Return all blue nodes that have a pink child"

$\exists y\ P_{\bigcirc}(x) \land P_{\bigcirc}(y) \land x \rightarrow y$

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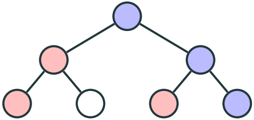

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Result: $\{ (x_1, \ldots, x_k) \mid (x_1, \ldots, x_k) \models Q \}$

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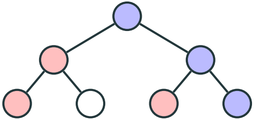

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Up to $|T|^k$ many answers

Enumeration algorithm

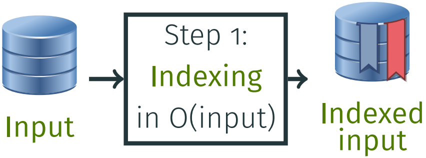
Input

Enumeration algorithm

Input →

Step 1:
Indexing
in O(input)

Enumeration algorithm



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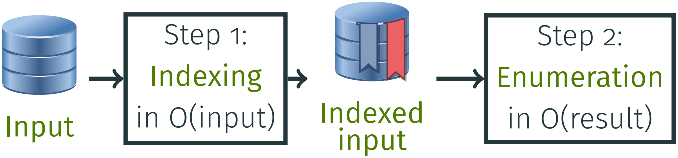

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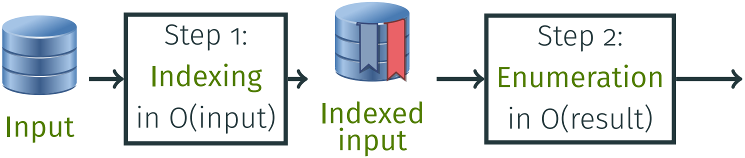
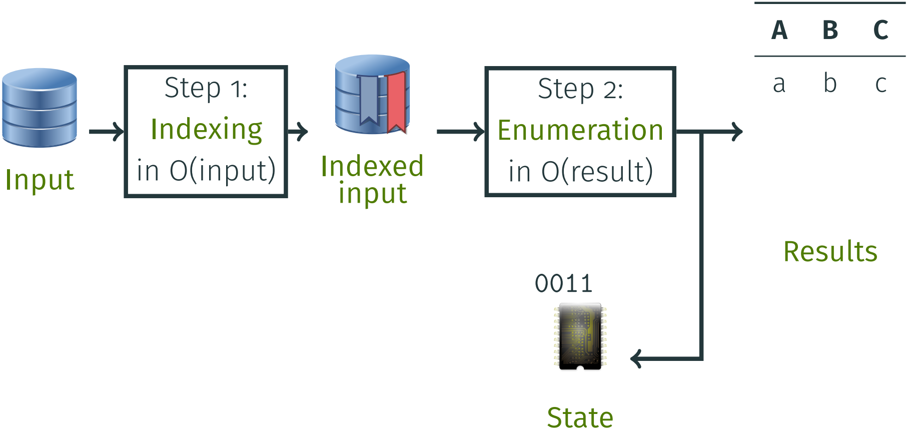


A	**B**	**C**
a	b	c

Results

Enumeration algorithm



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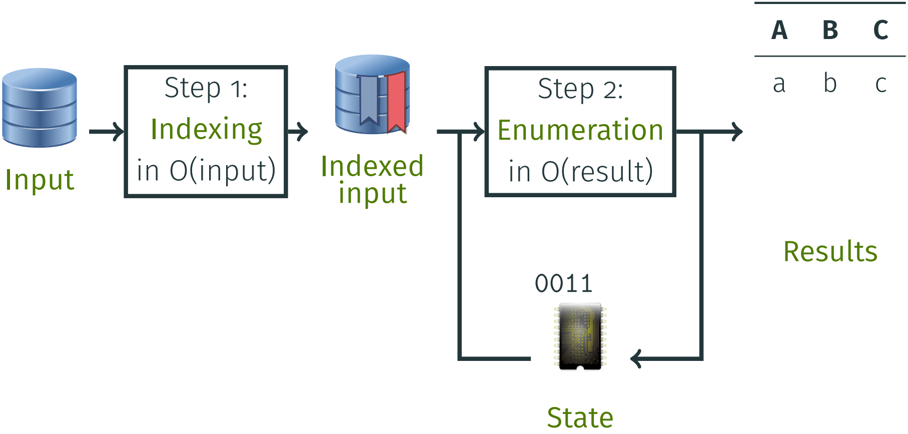

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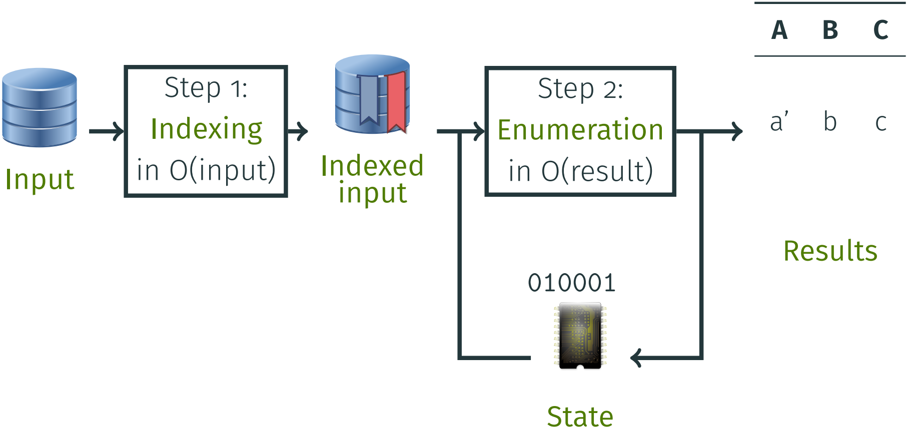

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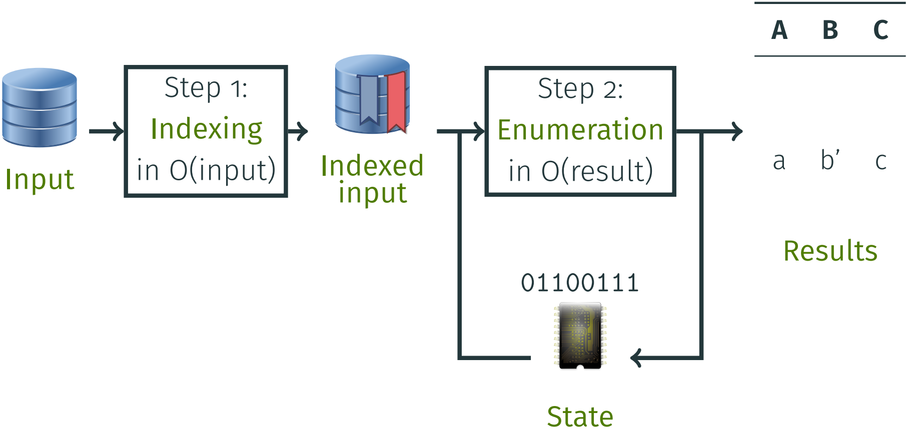

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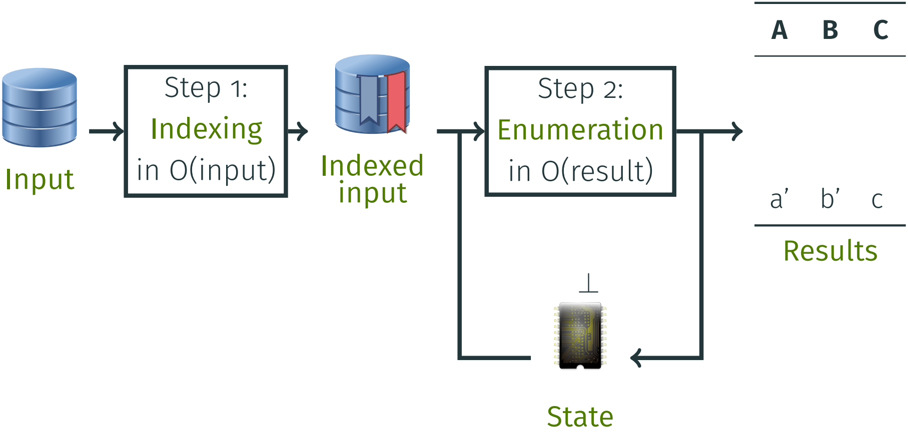

Known results on dynamic trees

All these results are on data complexity in T (for a fixed pattern):

Work	Data	Preproc.	Delay	Updates
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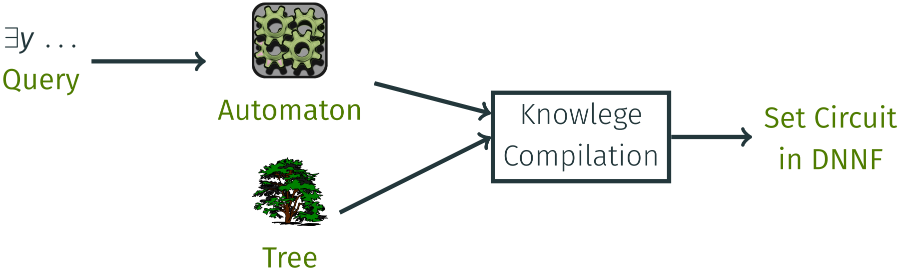
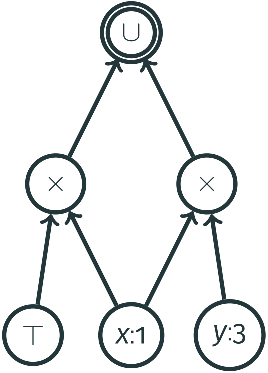


Semantics of set circuits



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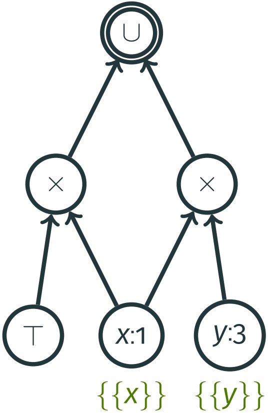

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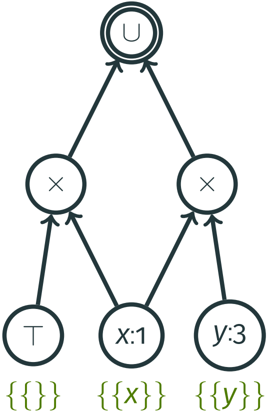

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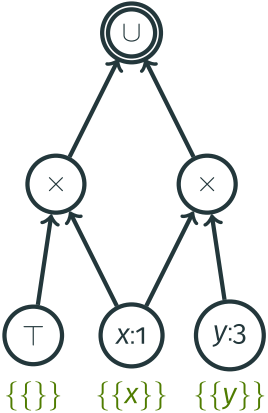

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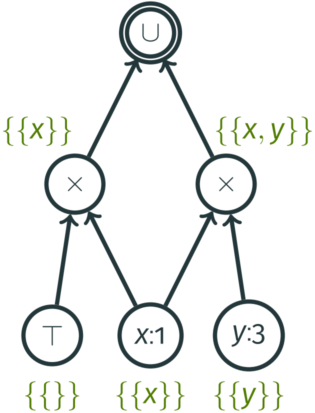

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$S(\text{(}\times\text{)}) := \{s_1 \cup s_2 \mid s_1 \in S(g_1), s_2 \in S(g_2)\}$

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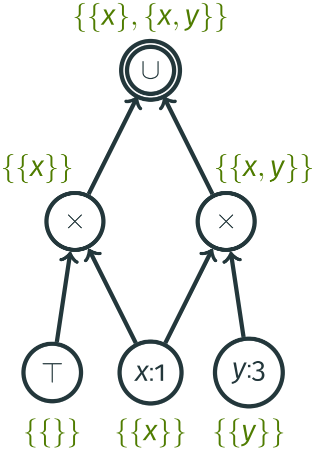

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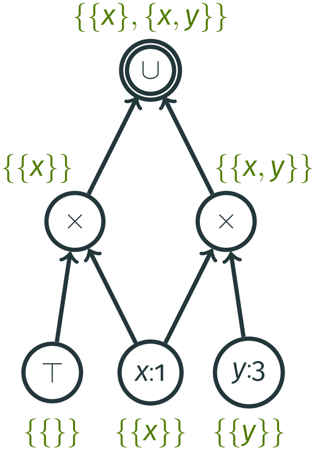

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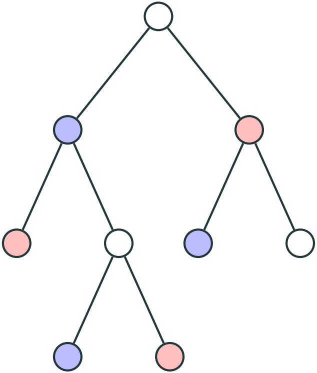
Task: Enumerate the elements of the set $S(g)$ captured by a gate g

$\rightarrow$E.g., for $S(g) = \{\{x\}, \{x, y\}\}$, enumerate $\{x\}$ and then $\{x, y\}$

Compiling Trees in Set Circuits



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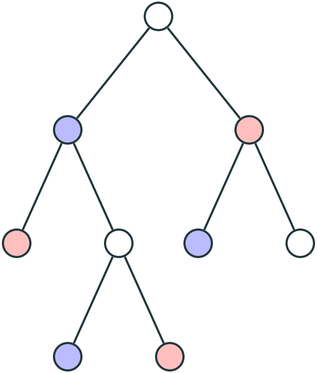

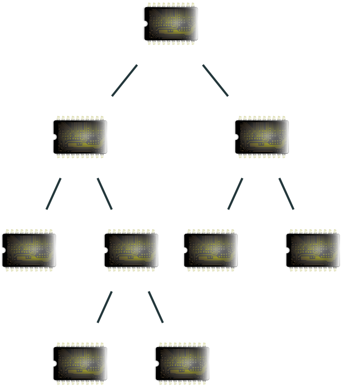

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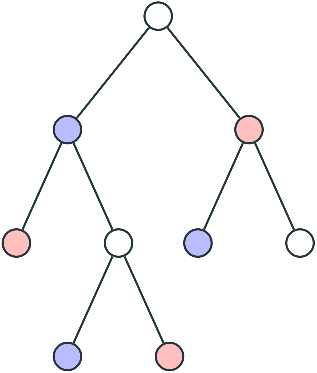

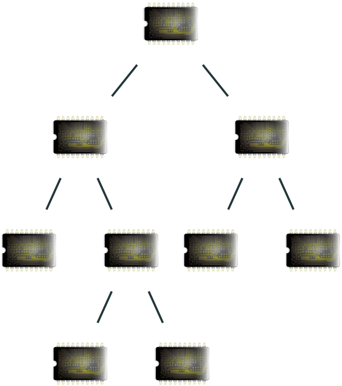

- One box for each node of the tree

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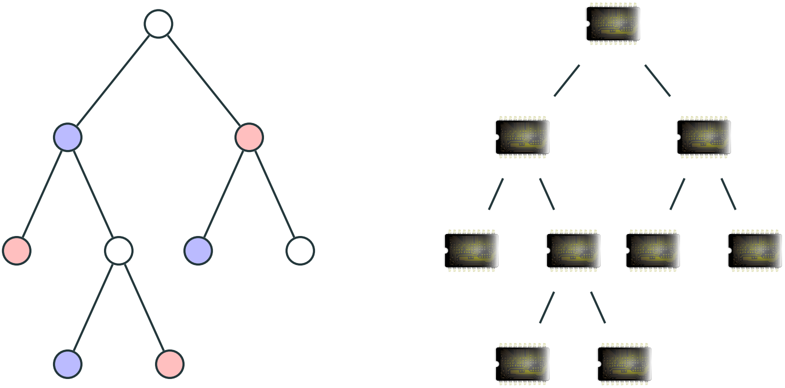
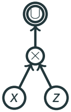


- One box for each node of the tree
- In each box: one ∪-gate for each state q of the automaton
  - Captures partial runs that end in q

Enumerate Circuit Results

Preprocessing phase:



DNNF
set circuit

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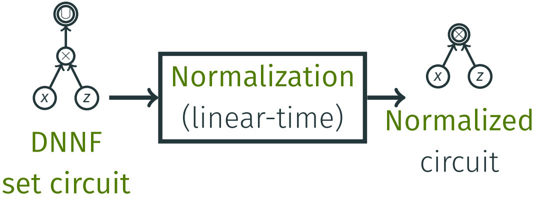

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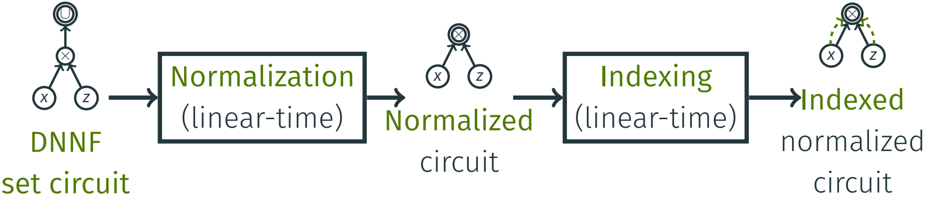

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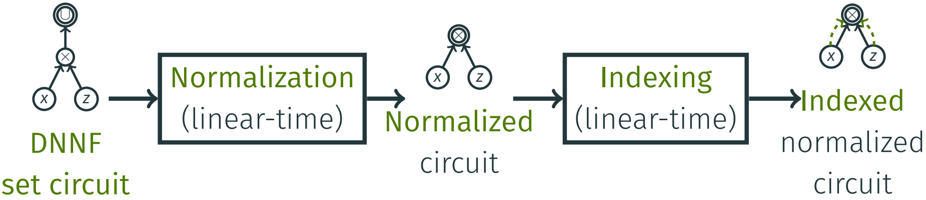

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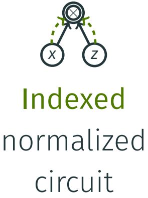

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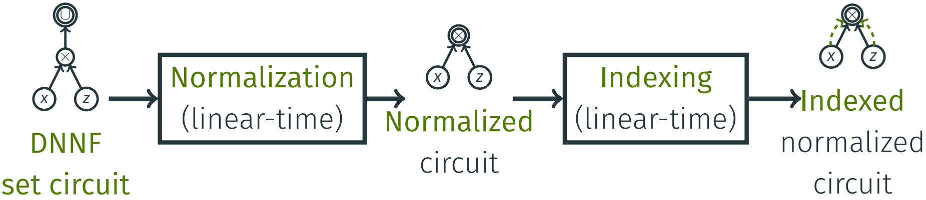

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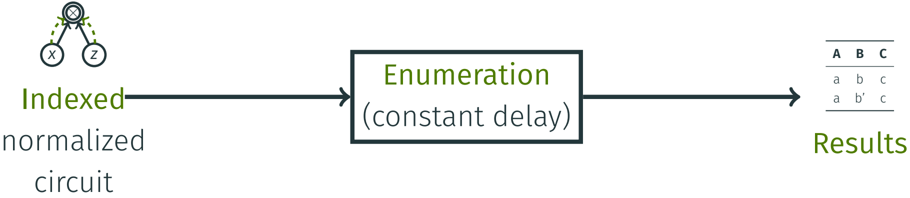

A	B	C
a	b	c
a	b′	c

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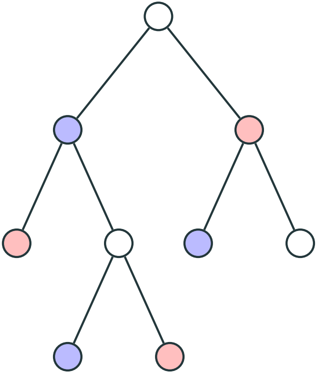

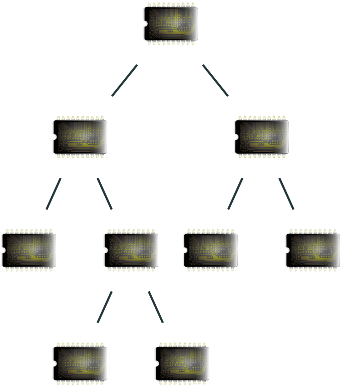

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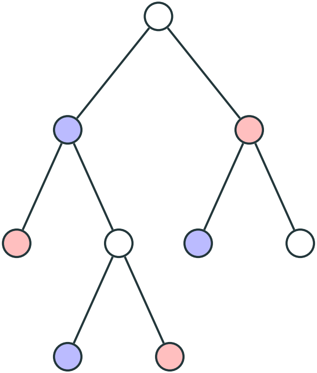

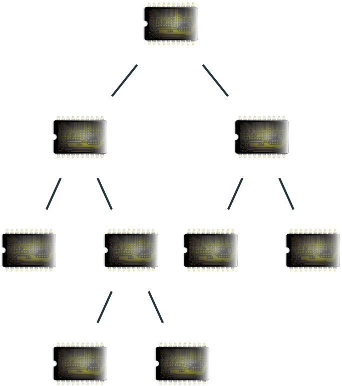

- Constructions are bottom-up

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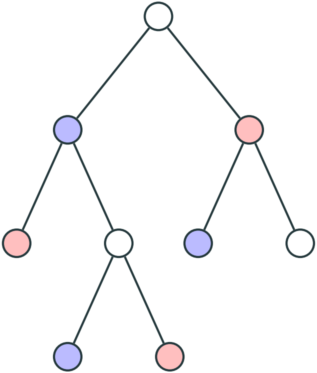

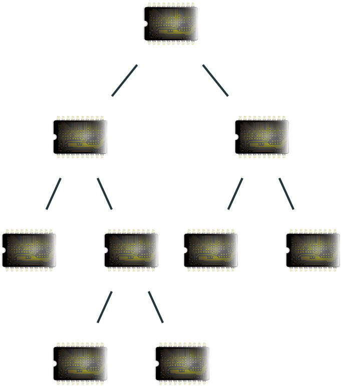

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- **Updates** can be done in $\mathcal{O}(\mathrm{depth}(T))$

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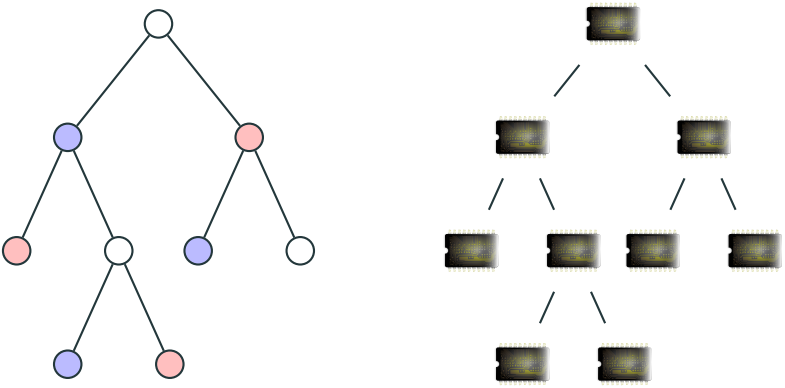

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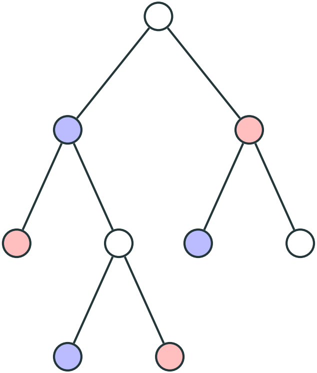

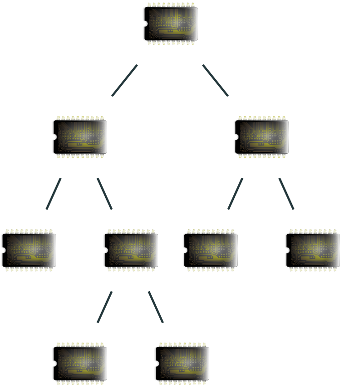

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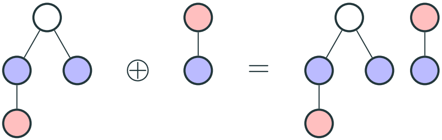
- Solution: Depict trees by forest algebra terms

Free Forest Algebra in a Nutshell



concatenation

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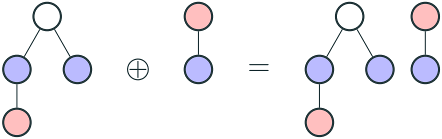

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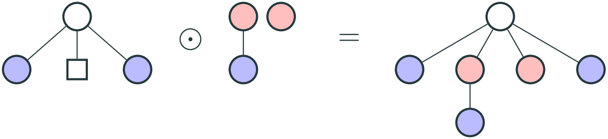

context
application

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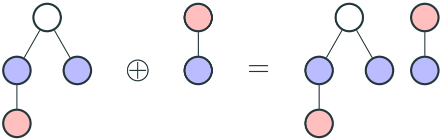

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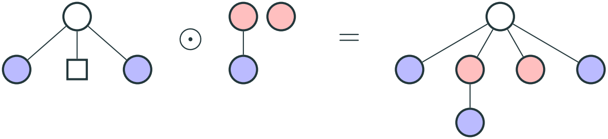

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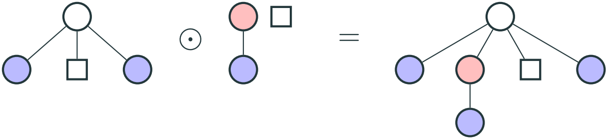

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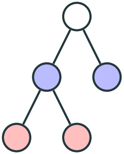

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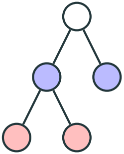
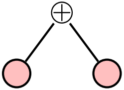
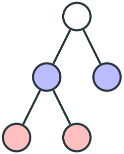


term



Free Forest Algebra in a Nutshell

tree



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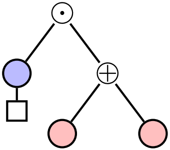

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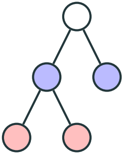

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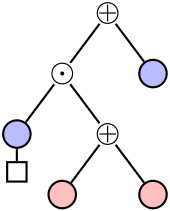
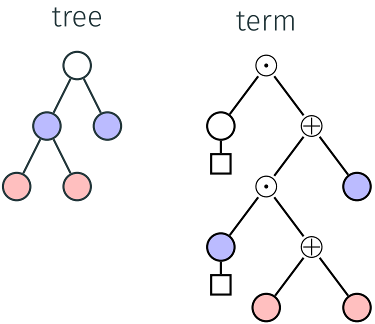


Free Forest Algebra in a Nutshell



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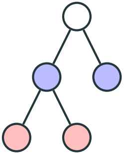

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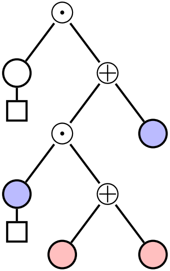
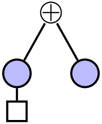


alternative term



Free Forest Algebra in a Nutshell

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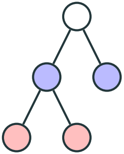

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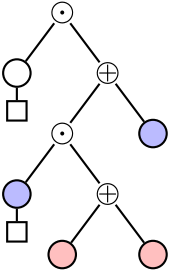

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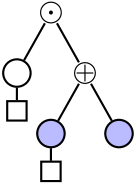
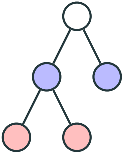


Free Forest Algebra in a Nutshell

tree



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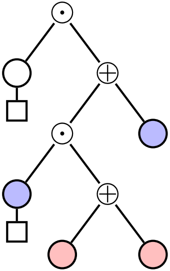

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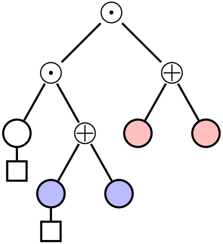
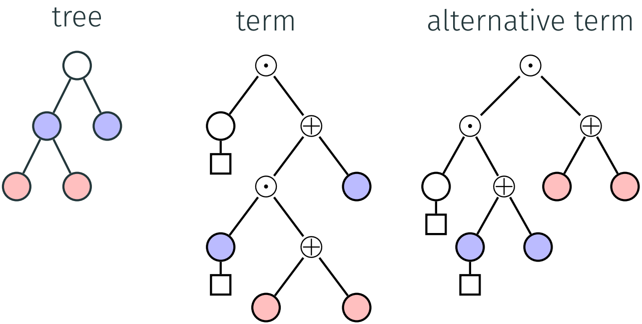
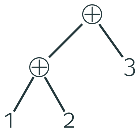


Free Forest Algebra in a Nutshell



The leaves of the formula correspond to the nodes of the tree

Rebalancing Forest Algebra Terms



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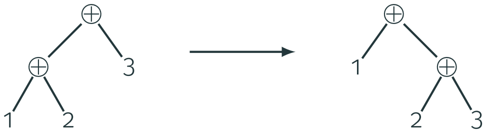

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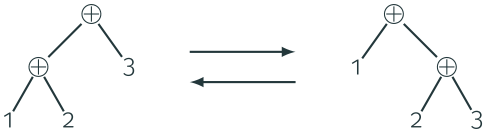

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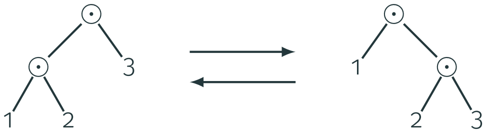

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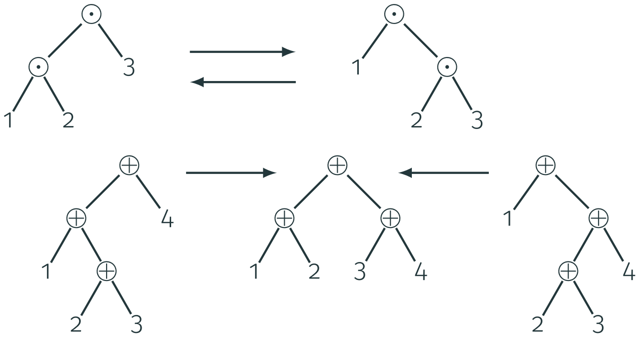

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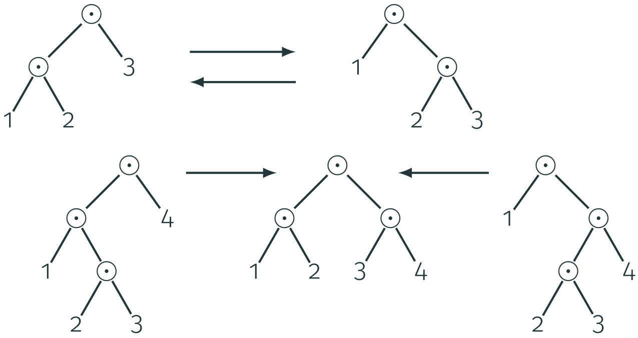

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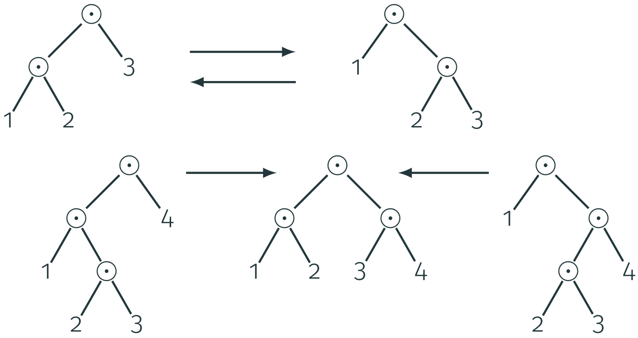

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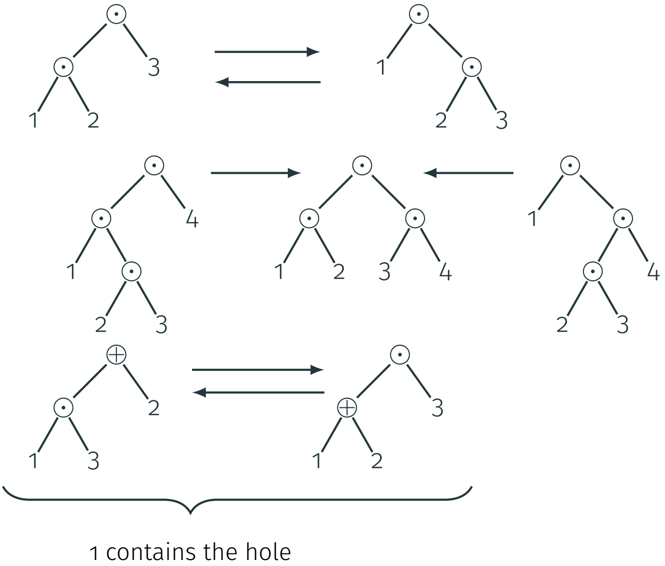
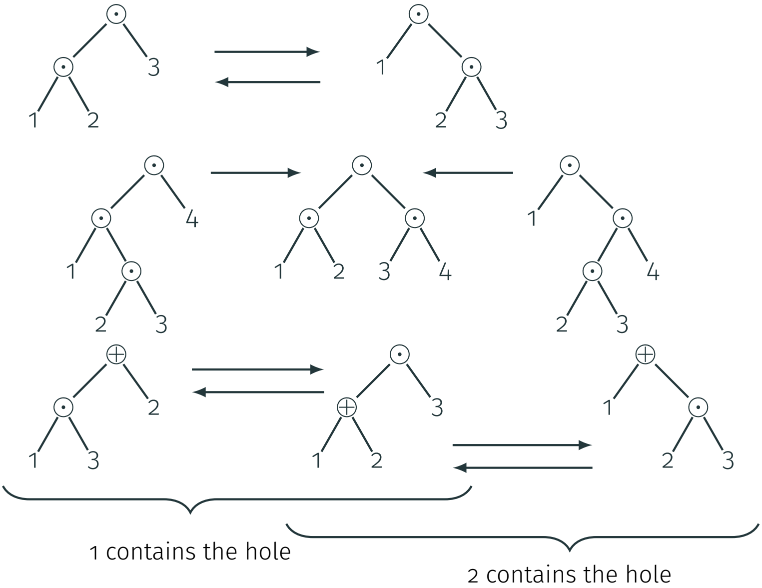


Rebalancing Forest Algebra Terms



Main Result

Theorem

Enumertion of MSO formulas on trees can be done in time:

Preprocessing	$O(\ |T| \times |Q|^{4\omega+1}\ )$
Delay	$O(\ |Q|^{4\omega} \times |S|\ )$
Updates	$O(\ \log(|T|) \times |Q|^{4\omega+1}\ )$

$|T|$	*size of tree*
$|Q|$	*number of states of a nondeterministic tree automaton*
$|S|$	*size of result*
ω	*exponent for Boolean matrix multiplication*

Lower Bound

Existencial Marked Ancestor Queries

Input: Tree t with some marked nodes

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Reduction to Query Enumeration with Updates

Fixed Query Q: Return all special nodes with a marked ancestor

For every marked ancestor query v:

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2. Enumerate Q and return "yes", iff Q produces some result
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Theorem: $\max(t_{\mathbf{delay}}, t_{\mathbf{update}}) \in \Omega\left(\frac{\log(n)}{\log\log(n)}\right)$

Results

Theorem

Enumertion of MSO formulas on trees can be done in time:

Preprocessing	$O(\ \lvert T\rvert \times \lvert Q\rvert^{4\omega+1}\ )$
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$$\max(t_{delay}, t_{update}) \in \Omega\left(\frac{\log(n)}{\log\log(n)}\right)$$

Thank You

References i

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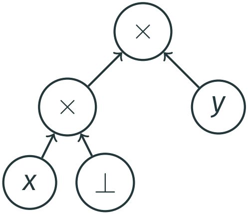
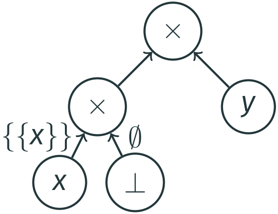
Normalization: handling $\emptyset$



Normalization: handling $\emptyset$



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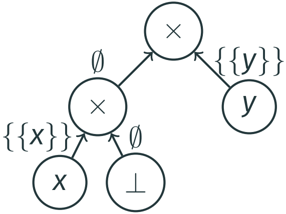

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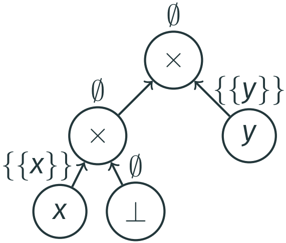

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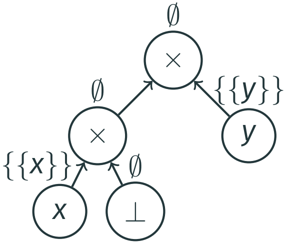

- Problem: if $S(g) = \emptyset$ we waste time

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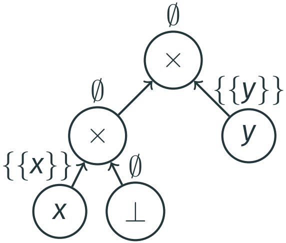

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  - compute bottom-up if $S(g) = \emptyset$

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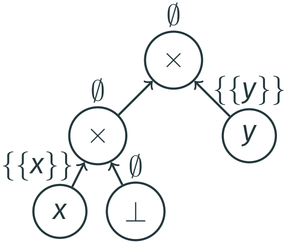
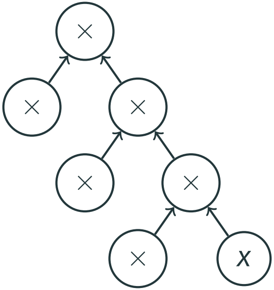


- Problem: if $S(g) = \emptyset$ we waste time
- Solution: in preprocessing
  - compute bottom-up if $S(g) = \emptyset$
  - then get rid of the gate

Normalization: handling empty sets



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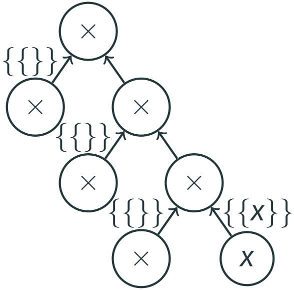

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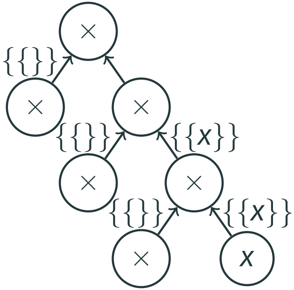

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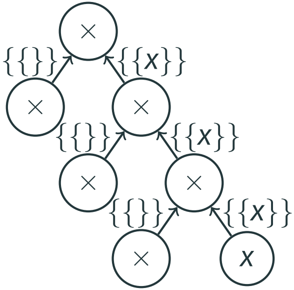

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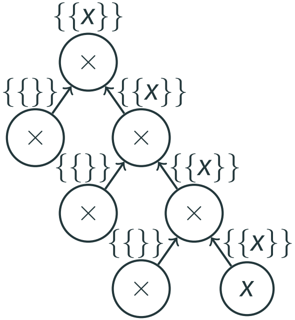

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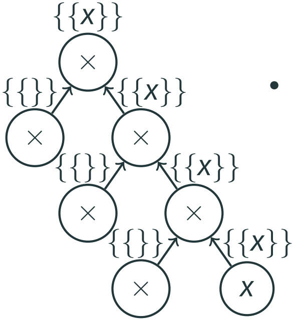
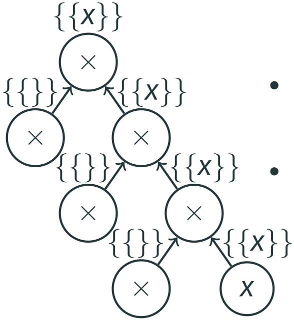


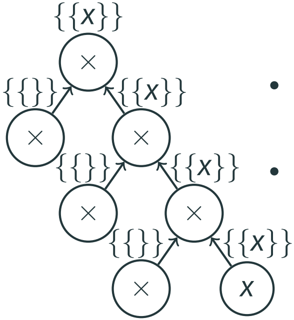
- Problem: if $S(g)$ contains $\{\}$ we waste time in chains of $\times$-gates

Normalization: handling empty sets



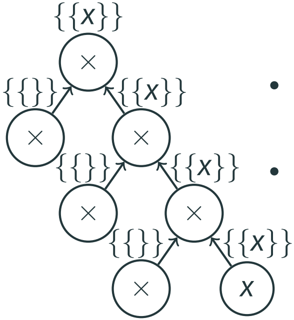
- Problem: if $S(g)$ contains {} we waste time in chains of ×-gates
- Solution:

Normalization: handling empty sets



- Problem: if $S(g)$ contains $\{\}$ we waste time in chains of $\times$-gates
- Solution:
  - remove inputs with $S(g) = \{\{\}\}$ for $\times$-gates

Normalization: handling empty sets



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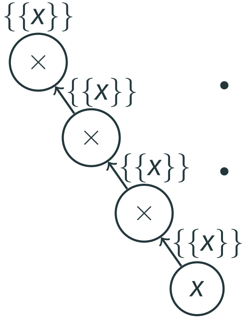

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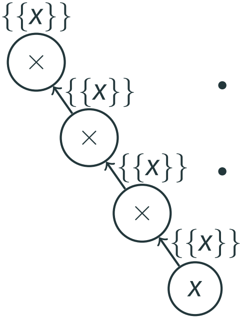

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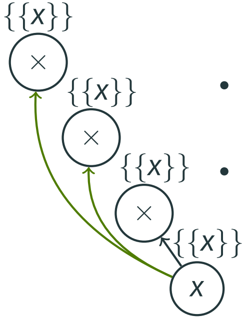

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- **Solution:**
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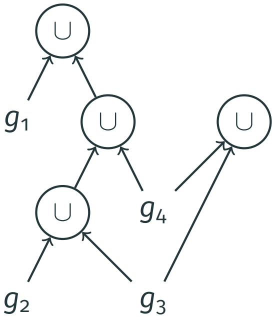
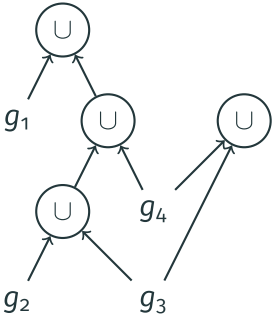
$\rightarrow$ Now, traversing a $\times$-gate ensures that we make progress: it splits the sets non-trivially

Indexing: handling ∪-hierarchies



- **Problem:** we waste time in ∪-hierarchies to find a **reachable exit** (non-∪ gate)

Indexing: handling ∪-hierarchies



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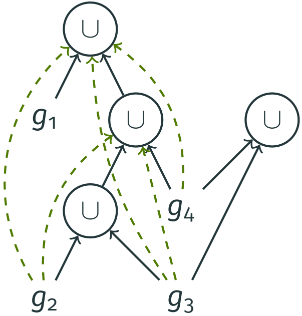

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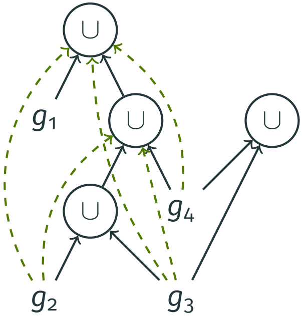

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- **Problem:** must be done in **linear time**

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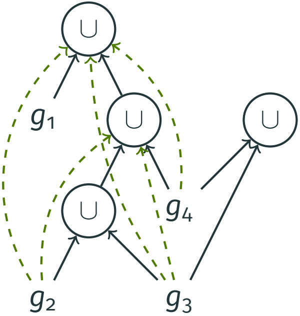

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- Solution: compute reachability index
- Problem: must be done in linear time

- Solution: Compute reachability index with box-granularity
- Use matrix multiplication
- Circuit has bounded width (by the size of the automaton)