Constant-Delay Enumeration for Nondeterministic Document Spanners

Antoine Amarilli[1], Pierre Bourhis[2], Stefan Mengel[3], **Matthias Niewerth**[4]

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[3]CNRS CRIL

[4]Universität Bayreuth

Problem: Finding Patterns in Text

- We have a long text T:

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→ How to find the pattern P efficiently in the text T?

Solution: Automata

- Convert the regular expression P to an automaton A

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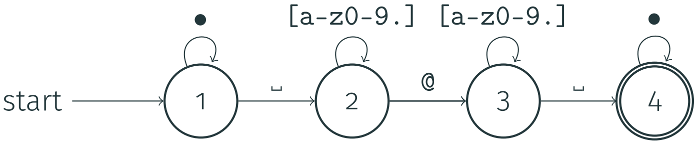

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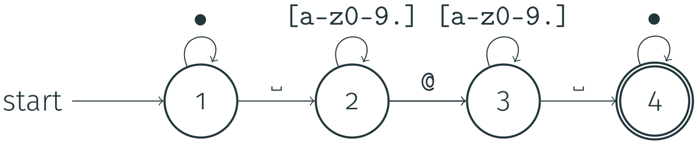

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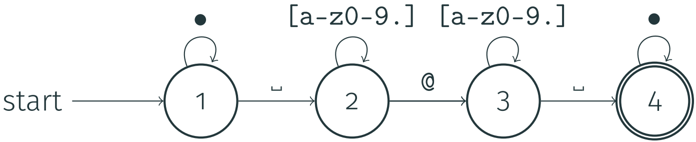

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`E m a i l ␣ a 3 n m @ a 3 n m . n e t ␣ A f f i l i a t i o n`

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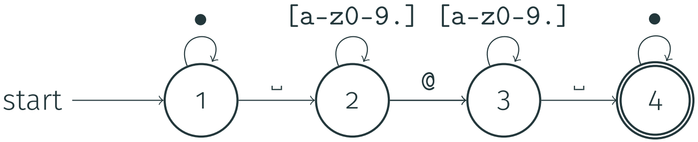

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`Email␣a3nm@a3nm.net␣Affiliation`

- The complexity is $O(|A| \times |T|)$, i.e., linear in T and polynomial in P
  - $\rightarrow$ This is very efficient in T and reasonably efficient in P

Actual Problem: Extracting all Patterns

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E	m	a	i	l	␣	a	3	n	m	@	a	3	n	m	.	n	e	t	␣	A	f	f	i	l	i	a	t	i	o	n

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$\rightarrow$ One match: $[5, 20\rangle$

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$$[186, 200\rangle, \quad [483, 500\rangle, \quad \ldots$$

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    - A sequential document spanner P given as a regular expression

$$P := \sqcup\ x\vdash \texttt{[a-z0-9.]}^* \ \texttt{@}\ \texttt{[a-z0-9.]}^* \dashv x\ \sqcup$$

  - Output: the list of tuples of substrings of T that match P:

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Measuring the Complexity

- Naive algorithm: Run the automaton A on each substring of T

l	o	l

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$[\rangle$ `l` `o` `l`

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$$\boxed{[\ \mathtt{l}\ \rangle\ \mathtt{o}\ \ \ \mathtt{l}}$$

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Measuring the Complexity

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```
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    ```
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    ```

  - Consider the **pattern** $P :=$ `a*`

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  - Consider the **pattern** $P :=$ `a*`
  - The **number of matches** is $\Omega(|T|^2)$

$\rightarrow$ We need a **different way** to measure complexity

Enumeration Algorithms

Idea: In real life, we do not want to compute all the matches
we just need to be able to enumerate matches quickly

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how to find patterns | **Search**

Results **1 - 20** of **10,514**

...

View (previous 20 | next 20) (20 | 50 | 100 | 250 | 500)

→ Formalization: enumeration algorithms

Formalizing Enumeration Algorithms

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Text T

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Phase 1:
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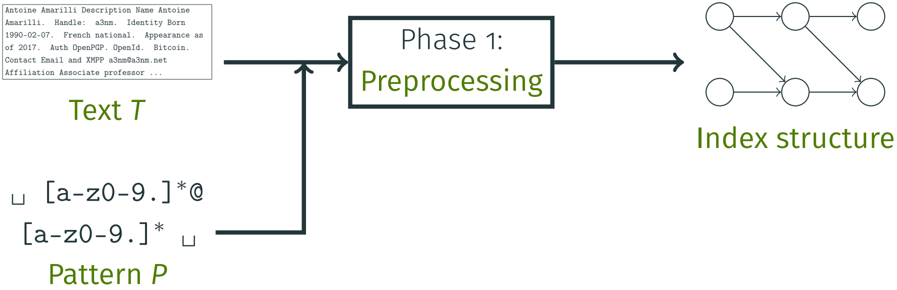

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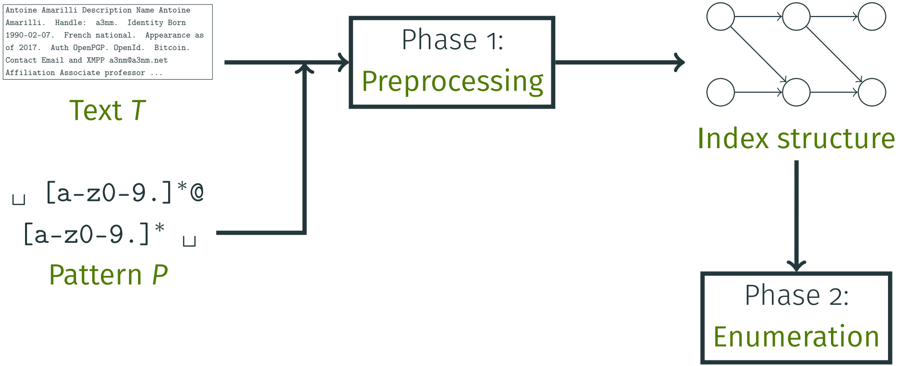

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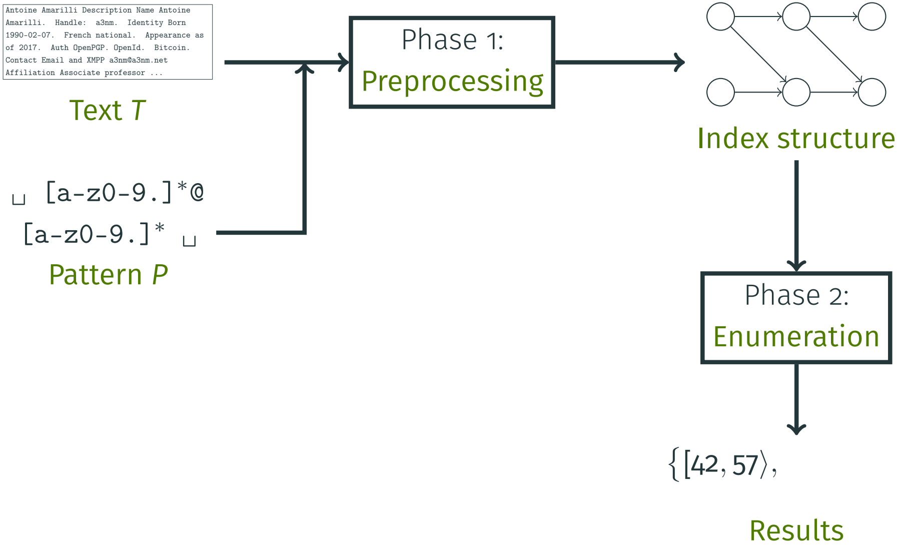

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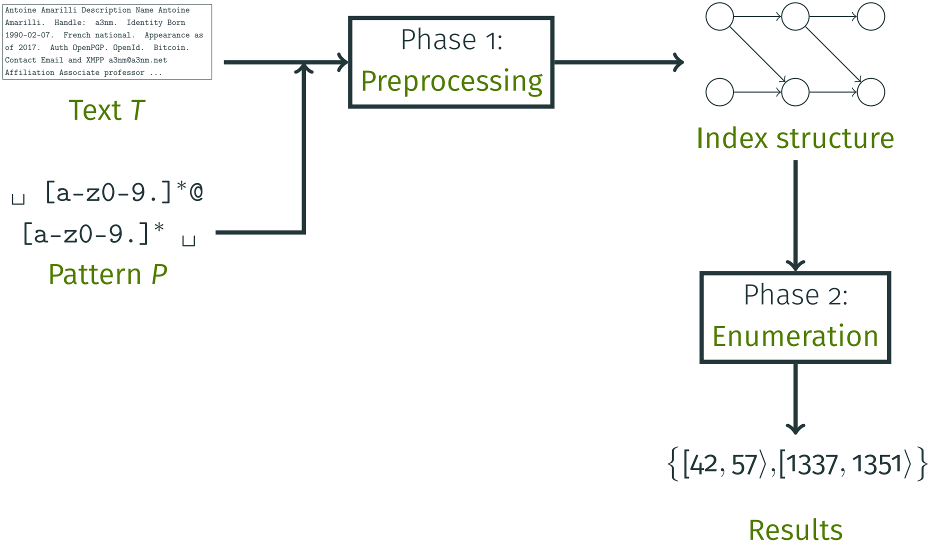

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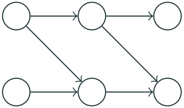

Index structure

Phase 2:
Enumeration

$\{[42, 57\rangle, [1337, 1351\rangle\}$

Results

Two performance criteria:

- **Total time** for phase 1
- **Delay between two results** in phase 2

... as a function of the **text** and **pattern**

Complexity of Enumeration Algorithms

- Recall the inputs to our problem:
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- What is the delay of the naive algorithm?

  $\rightarrow$ it is the maximal time to find the next matching substring

Complexity of Enumeration Algorithms

- Recall the **inputs** to our problem:
  - A **text** T

```
Antoine Amarilli Description Name Antoine Amarilli.  Handle:  a3nm.  Identity Born 1990-02-07.
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a3nm@a3nm.net Affiliation Associate professor of computer science (office C201-4) in the DIG team of
Télécom ParisTech, 46 rue Barrault, F-75634 Paris Cedex 13, France.  Studies PhD in computer science
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```

  - A **pattern** P given as a regular expression

$$P := \sqcup\ \texttt{[a-z0-9.]}^*\ \texttt{@}\ \texttt{[a-z0-9.]}^*\ \sqcup$$

- What is the **delay** of the **naive algorithm**?

$\rightarrow$ it is the **maximal time** to find the next **matching substring**

$\rightarrow$ i.e. $O(|T|^2 \times |A|)$, e.g., if only the **beginning** and **end** match

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→ Can we do **better**?

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Theorem

We can enumerate all matches of a pattern P on a text T with:

- *Preprocessing in $O(|T| \times Poly(P))$*
- *Delay polynomial in P and independent from T*

Automaton Formalism

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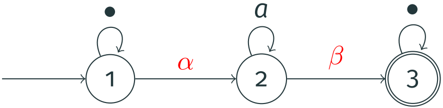

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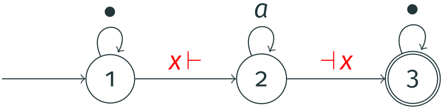

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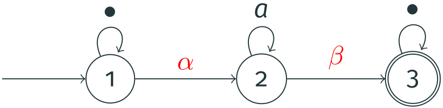

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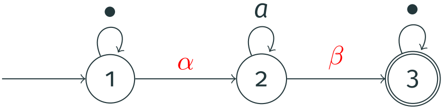

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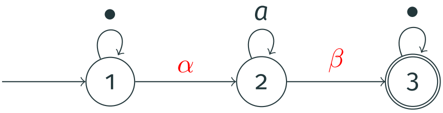

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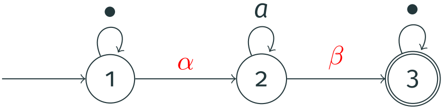

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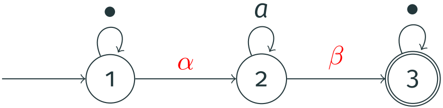

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- Assumption: There is no run for which A reads the same capture variable twice at the same position
- Challenge: Because of nondeterminism we can have many different runs of A producing the same tuple!

Proof idea: Product DAG

Compute a product DAG of the text T and of the automaton A

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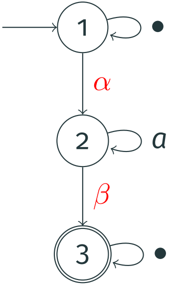

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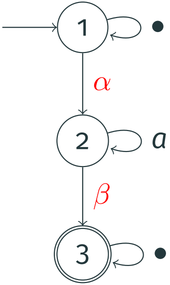
a	a	a	b	a



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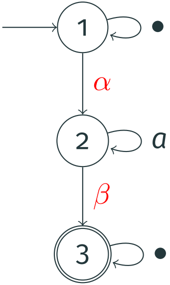

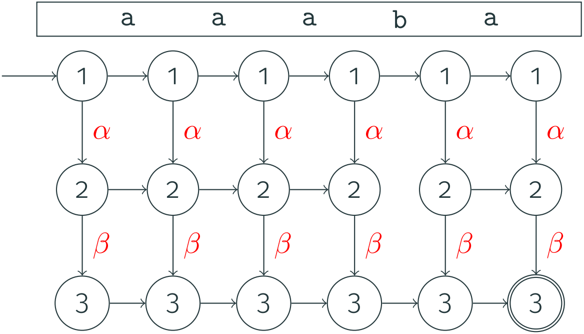

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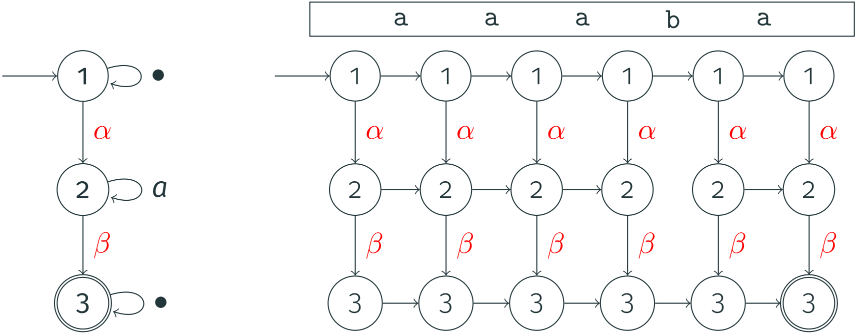

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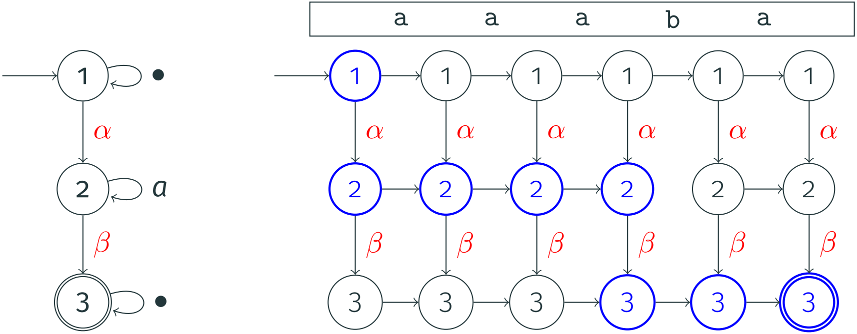
Example: Text $T :=$ `aaaba` and $P := \bullet^* \;\alpha\; a^* \;\beta\; \bullet^*$, match $\langle \alpha : 0, \beta : 3 \rangle$



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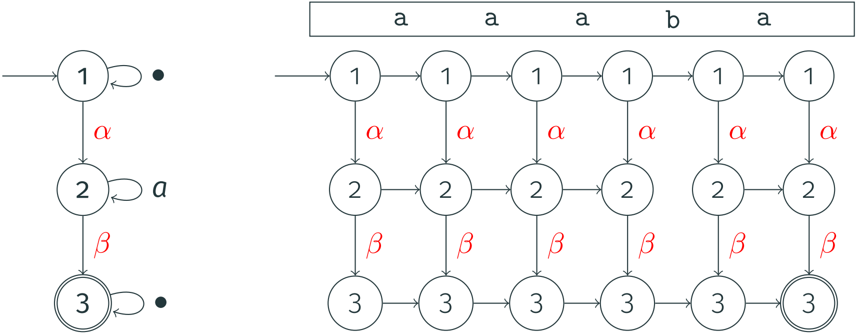

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→ Challenge: Enumerate paths but avoid duplicate matches
and do not waste time to ensure constant delay

Proof ingredients

Several ingredients to do this efficient

- Prune non-accepting paths
- Use shortcuts (pointers) to skip long paths
- Flashlight search

Proof ingredient: jump pointers to save time

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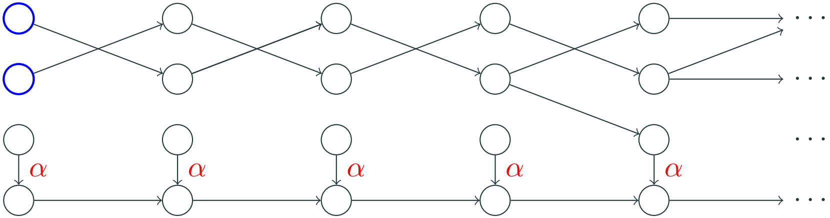

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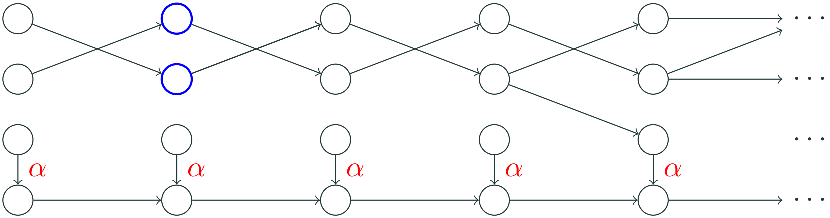

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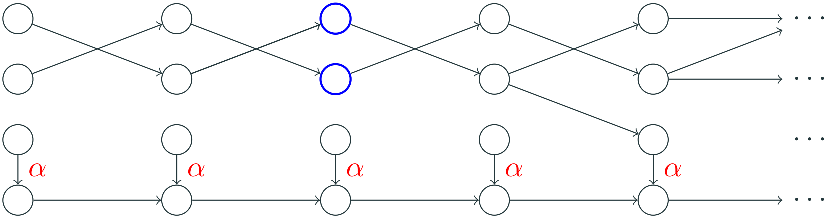

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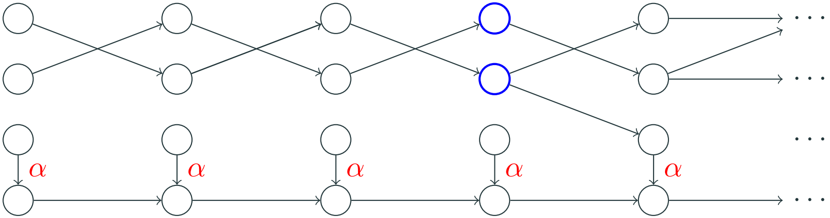

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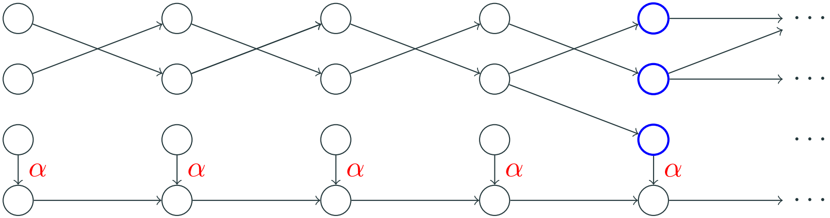

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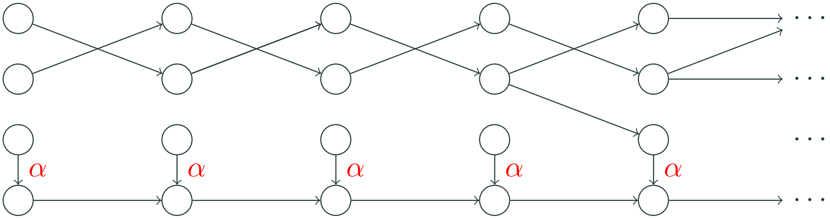

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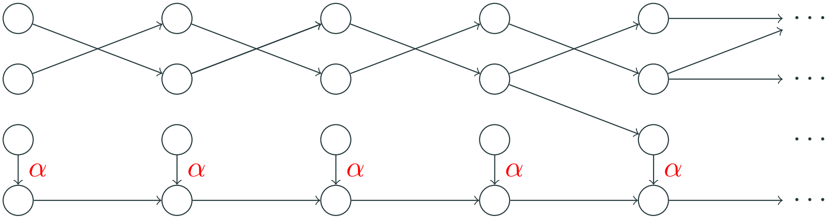

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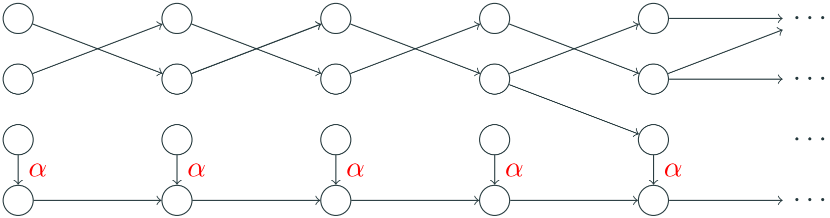

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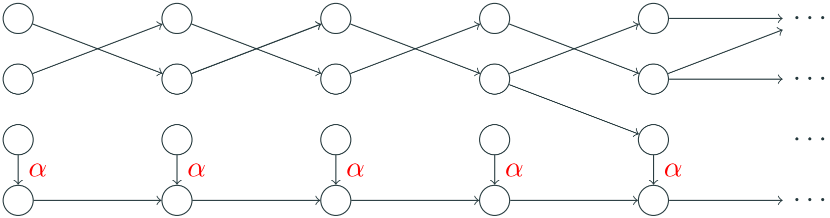

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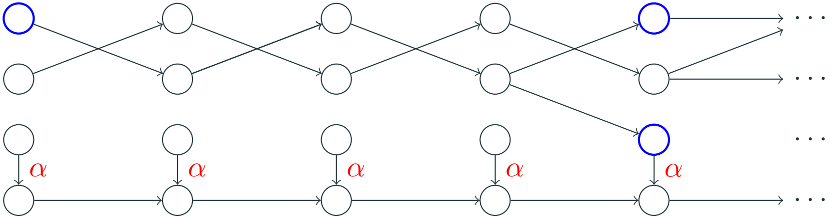

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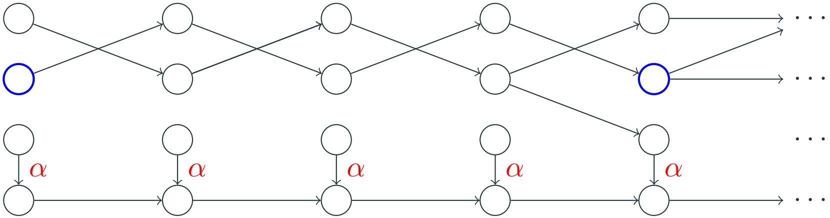

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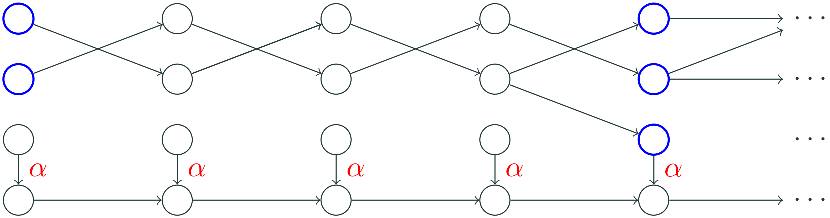

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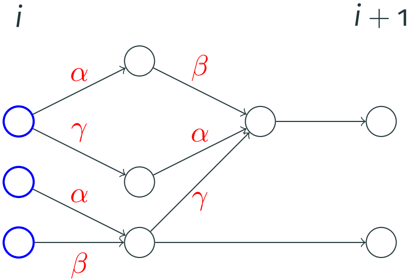

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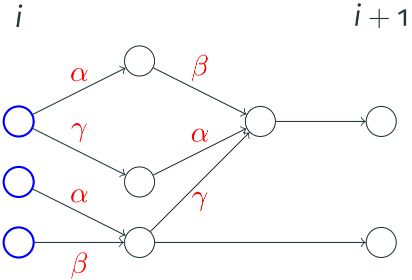

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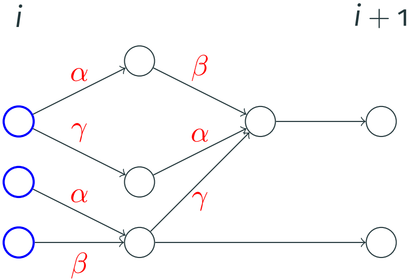

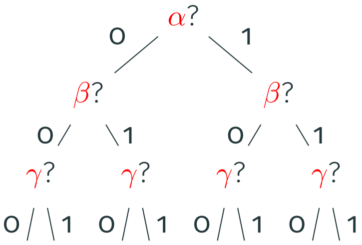

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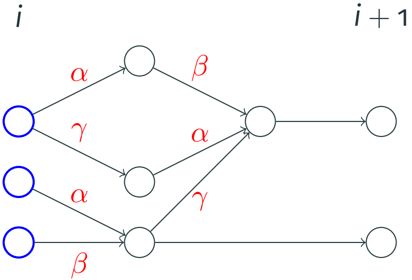

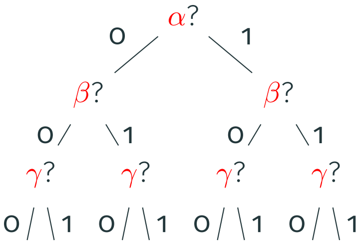

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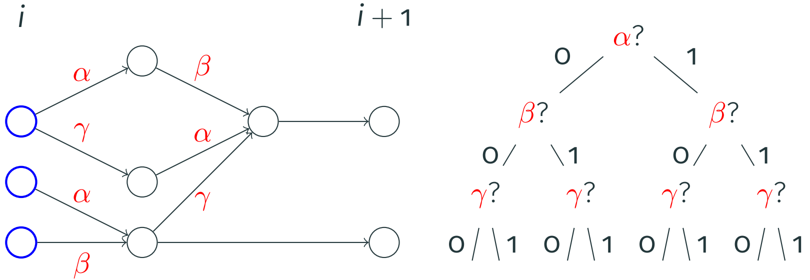

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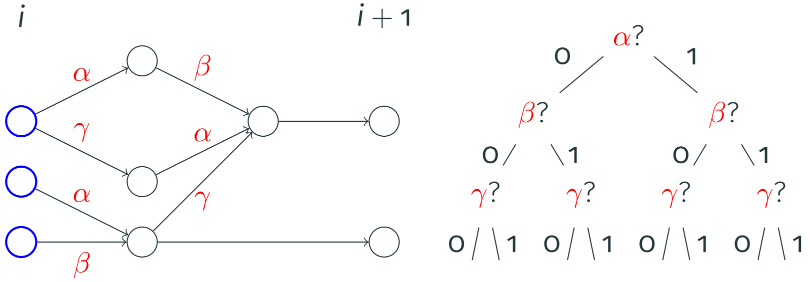

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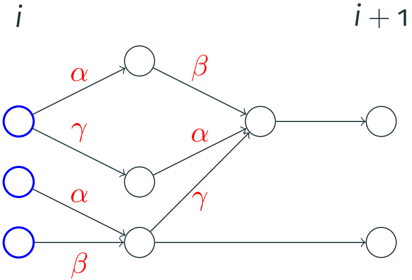

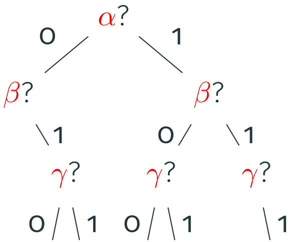

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Summary and Future Work

Main Result and Future Work

Theorem

Given a sequential document spanner P and text T, we can enumerate with:

$$\begin{aligned} \textit{Preprocessing} \quad & O(|P|^{\omega+1} \times |T|) \\ \textit{Delay} \quad & O(|\mathcal{V}^3| \times |P|^2) \end{aligned}$$

$\mathcal{V}$: Set of Variables

ω : Exponent for Boolean matrix multiplication

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$$\left.\begin{array}{l} \bullet\ \text{Extending the results from text to trees} \\ \bullet\ \text{Supporting updates on the input data} \end{array}\right\}\ \text{PODS 2019}$$

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Thanks for your attention!

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