Separability by Short Subsequences and Subwords

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Motivation

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What does a database theoretician do in the morning?

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- Get some coffee (optional)

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(It's a pretty complicated query, tweaked to your personal interests)

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- One day, something strange happens
- The query returns someone you didn't expect
- So you wonder: "What's going on here?"

Motivation

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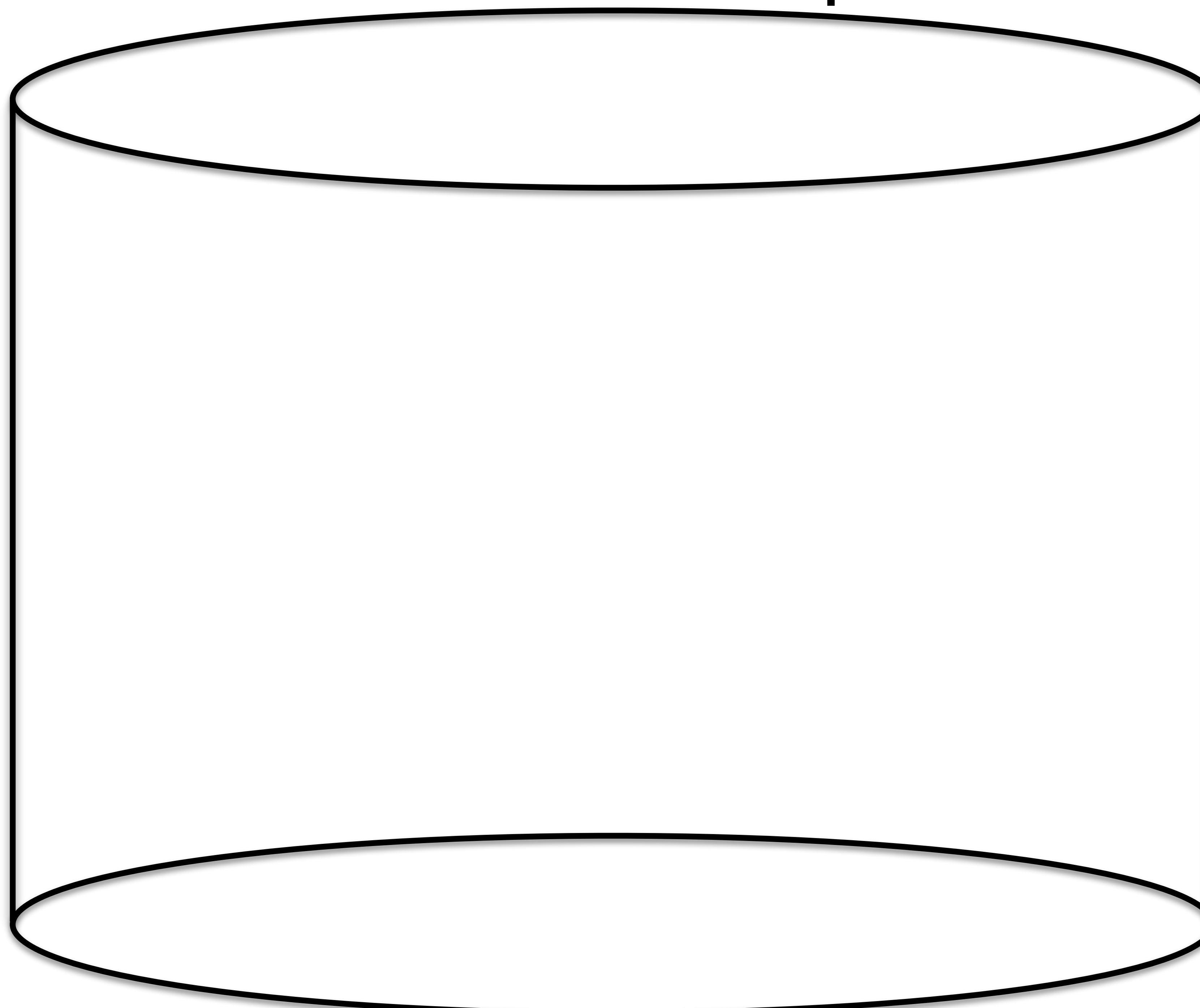
Graph Database



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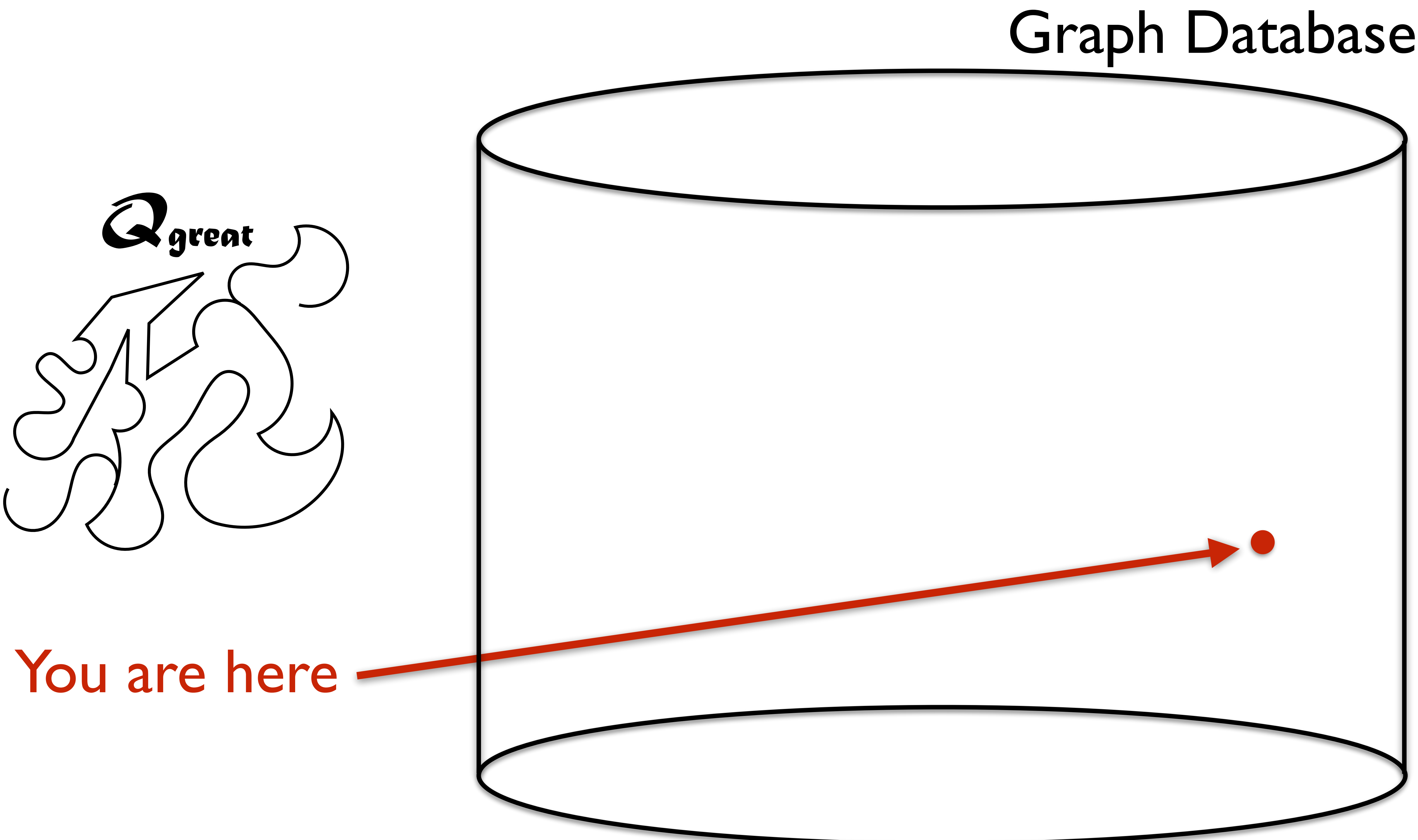

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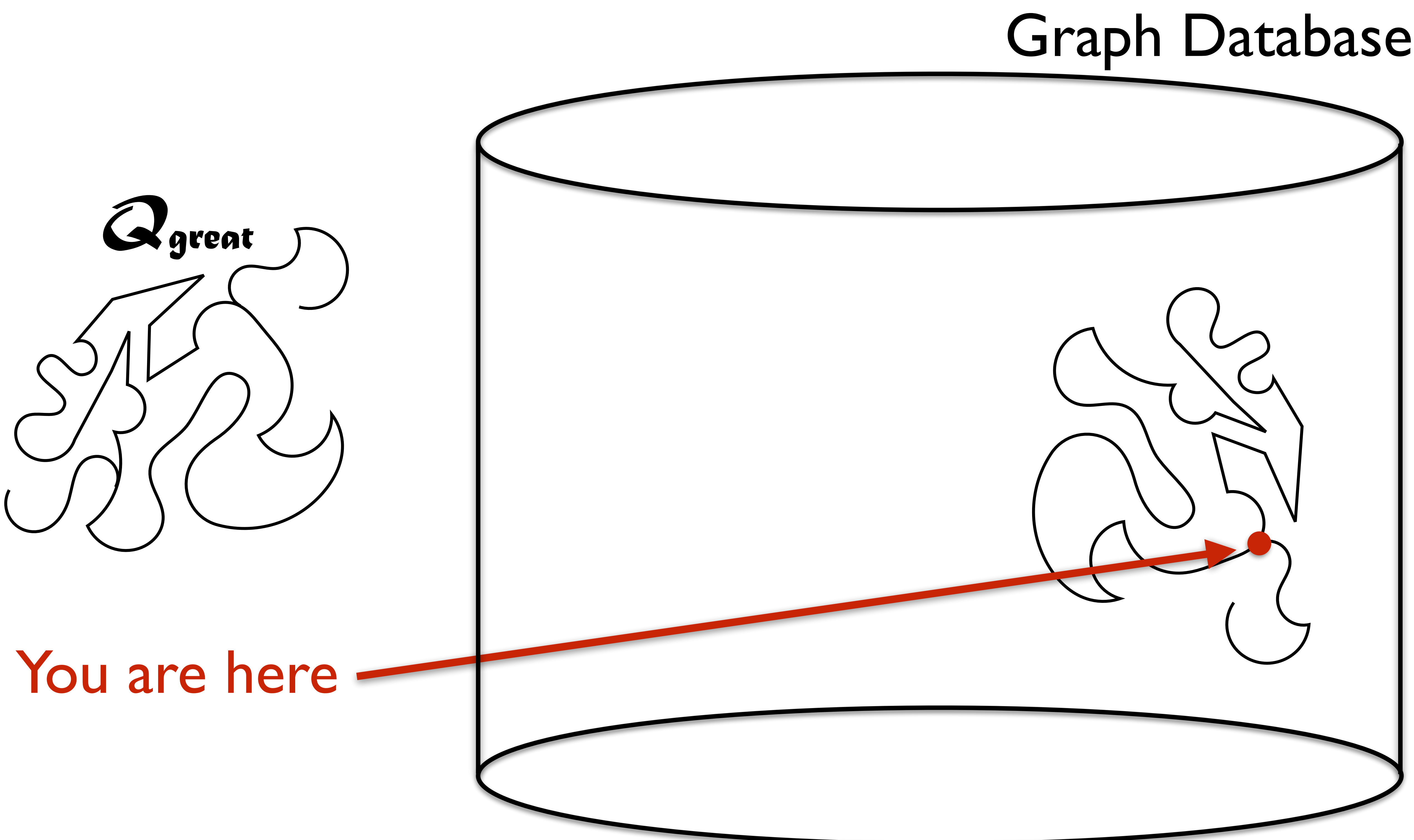

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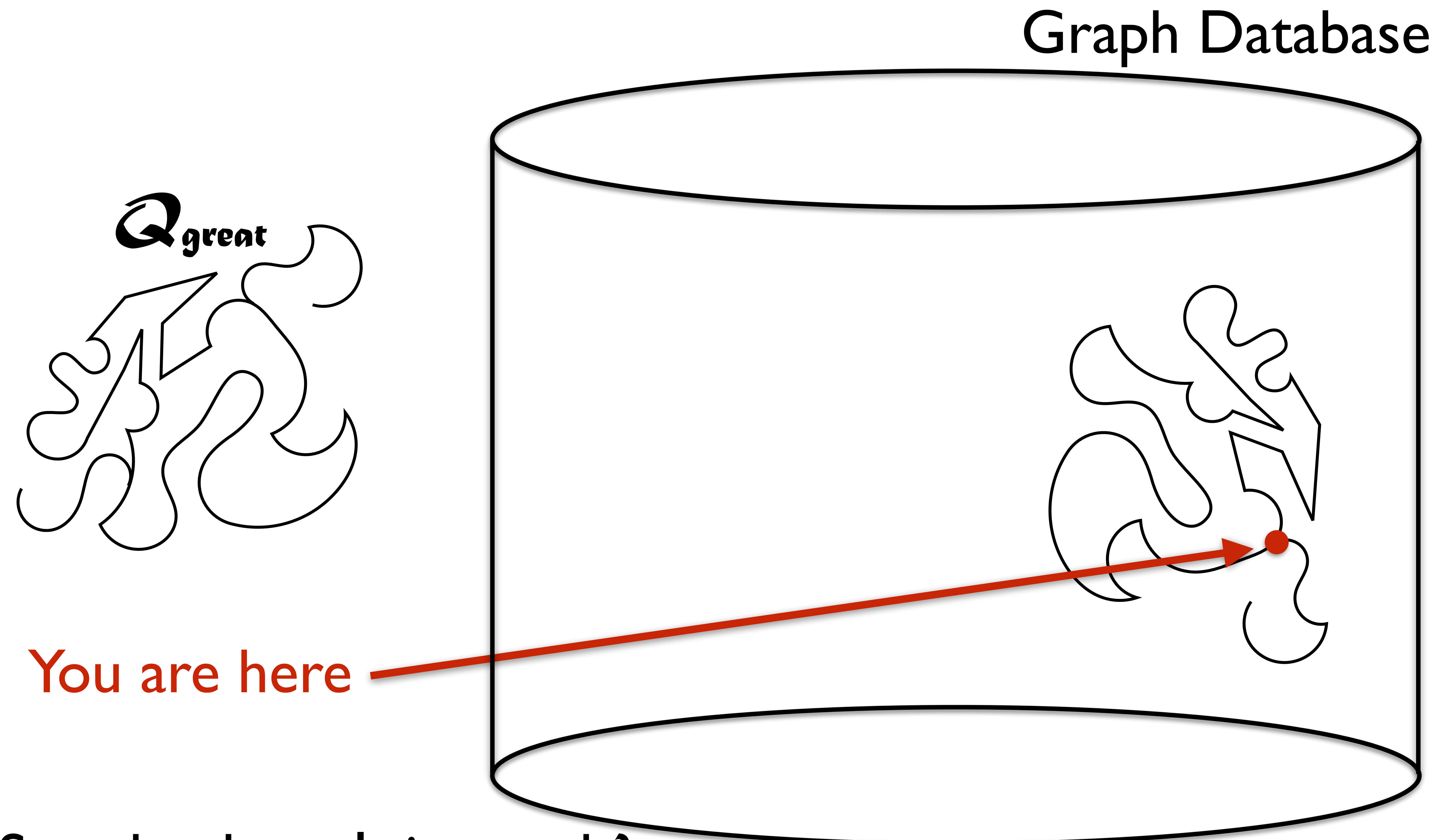

So, why doesn't it match?

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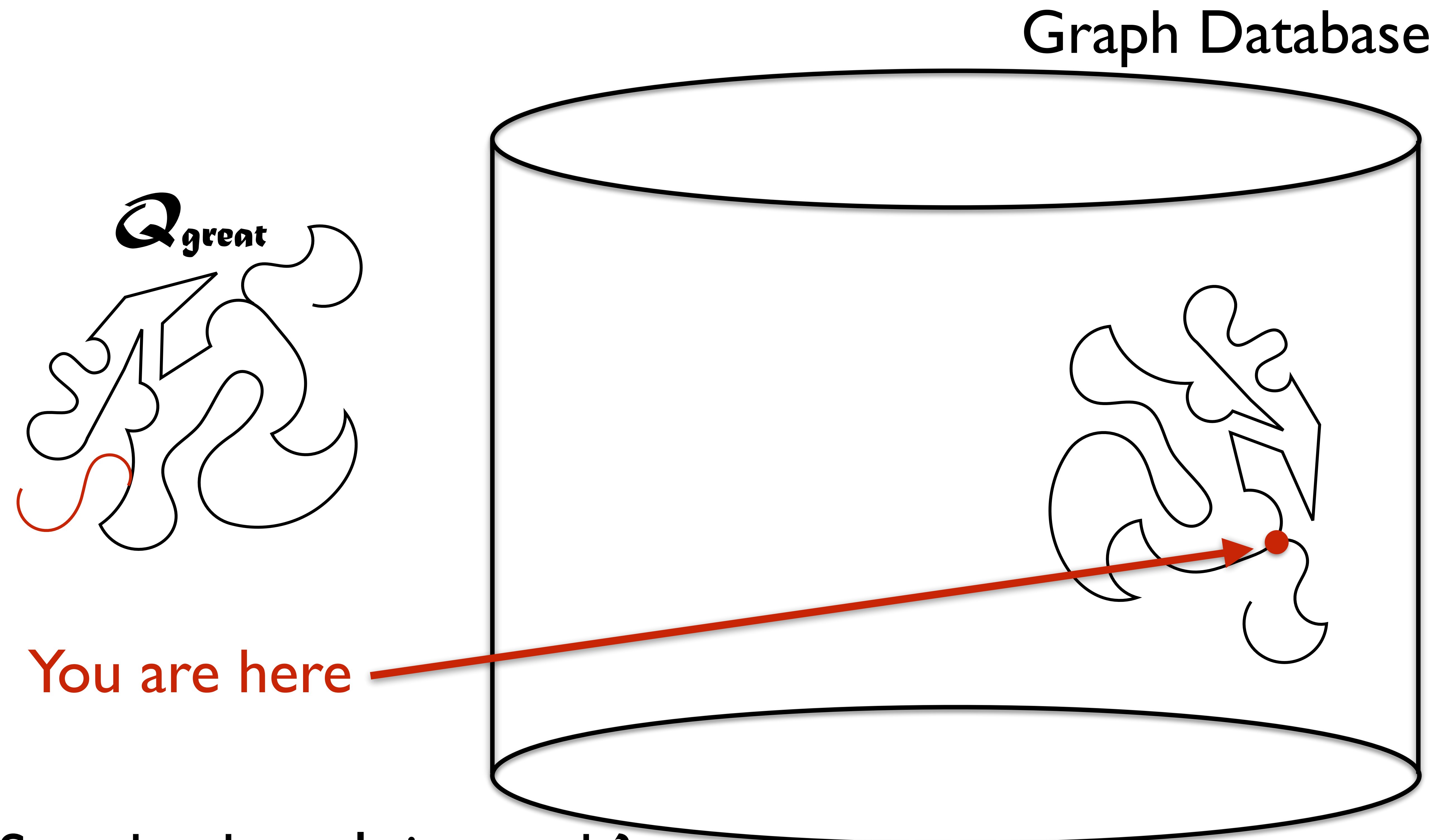

So, why doesn't it match?
Here, there's a very simple explanation

So, we're looking for

explanations
of
why a query
doesn't return a result you expect

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How do we formalize this?

More concrete

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The data is an edge-labeled directed graph G
The query is a Regular Path Query (RPQ) r

(regular expression)

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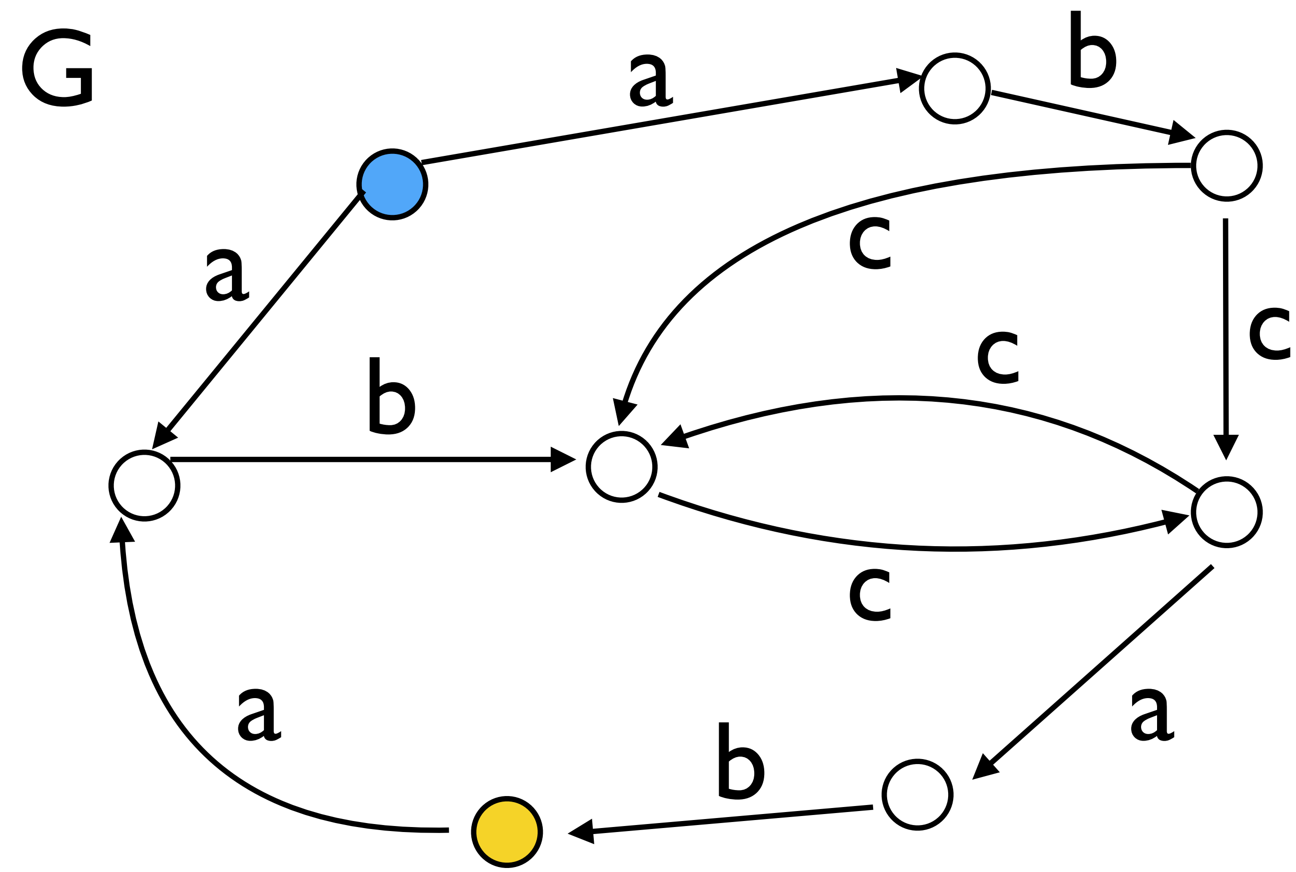
An RPQ r returns
pairs of nodes (x,y)
such that there is a path from x to y in G
that is labeled by a word in L(r)

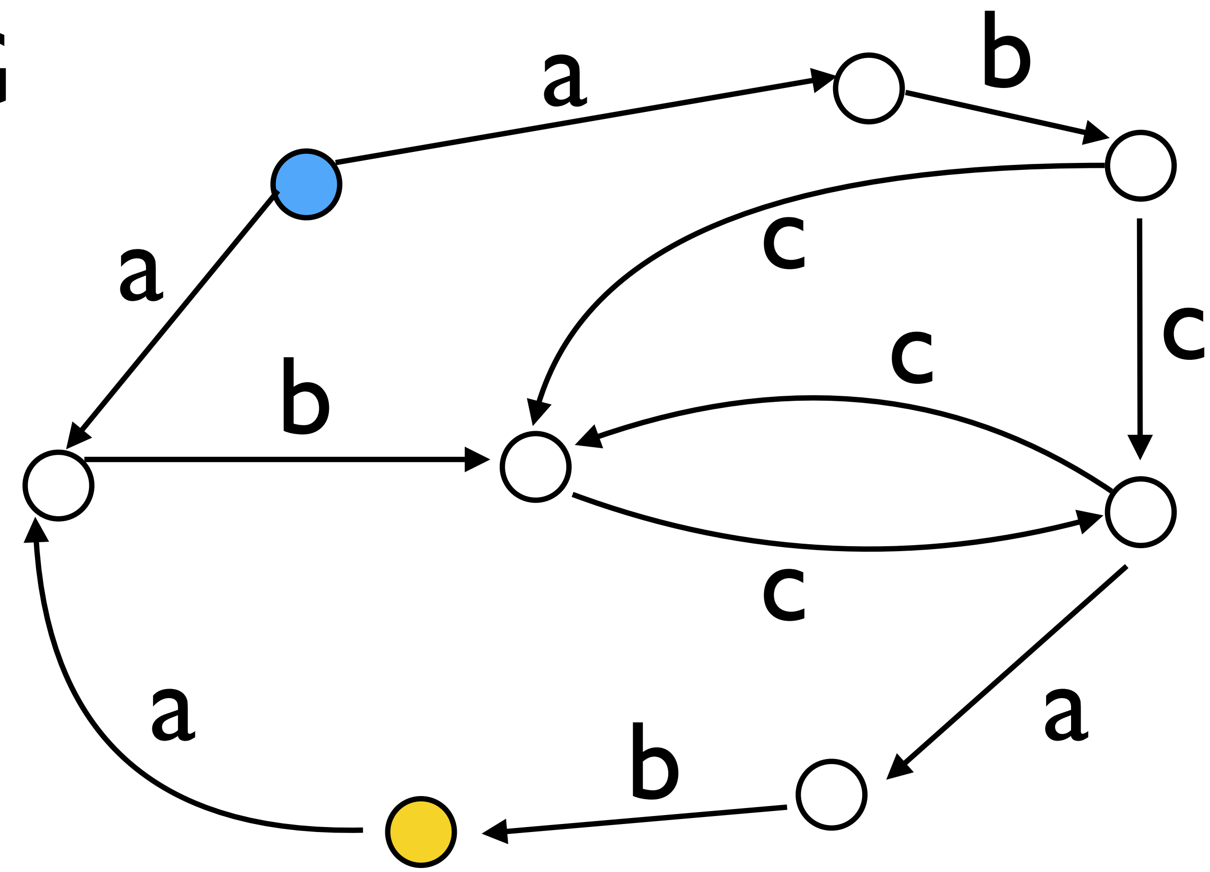
More concrete



r = ab(cc)*ab

More concrete

G



r = ab(cc)*ab

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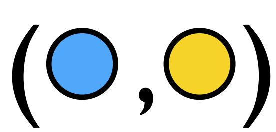

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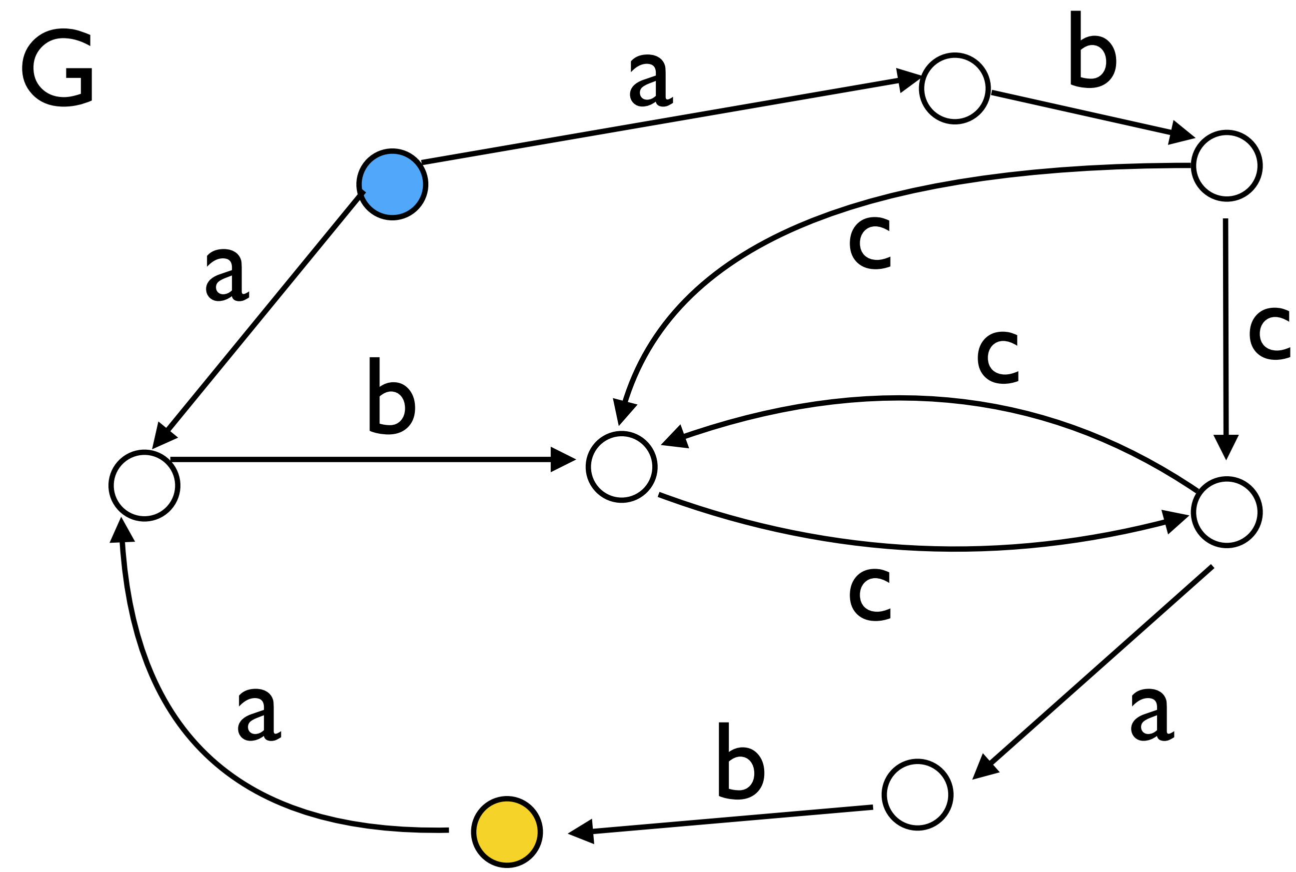


$r = ab(cc)^*ab$

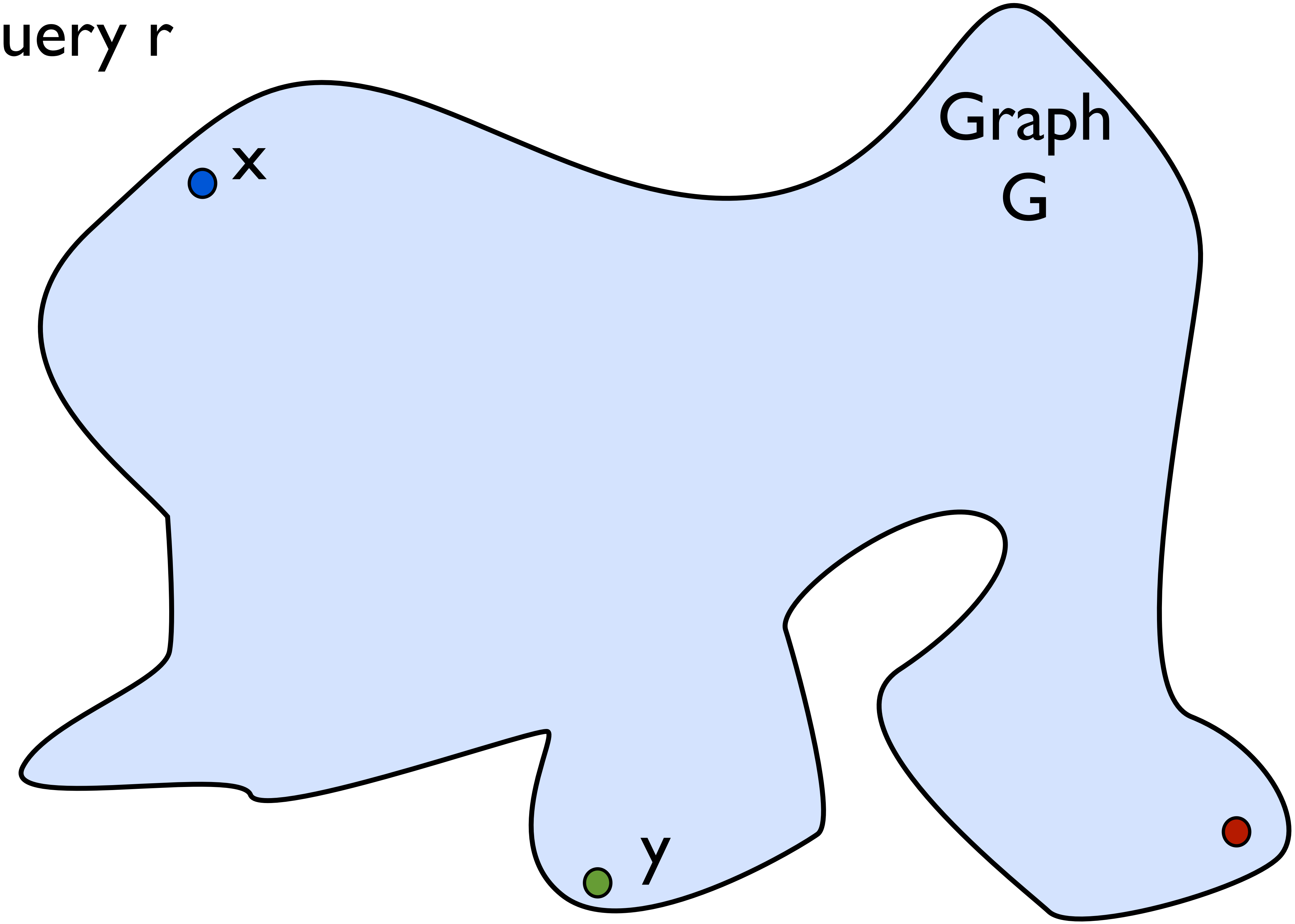
returns

(,)

but not (,)

More concrete

Reg Path Query r



More concrete

Reg Path Query r

Selects
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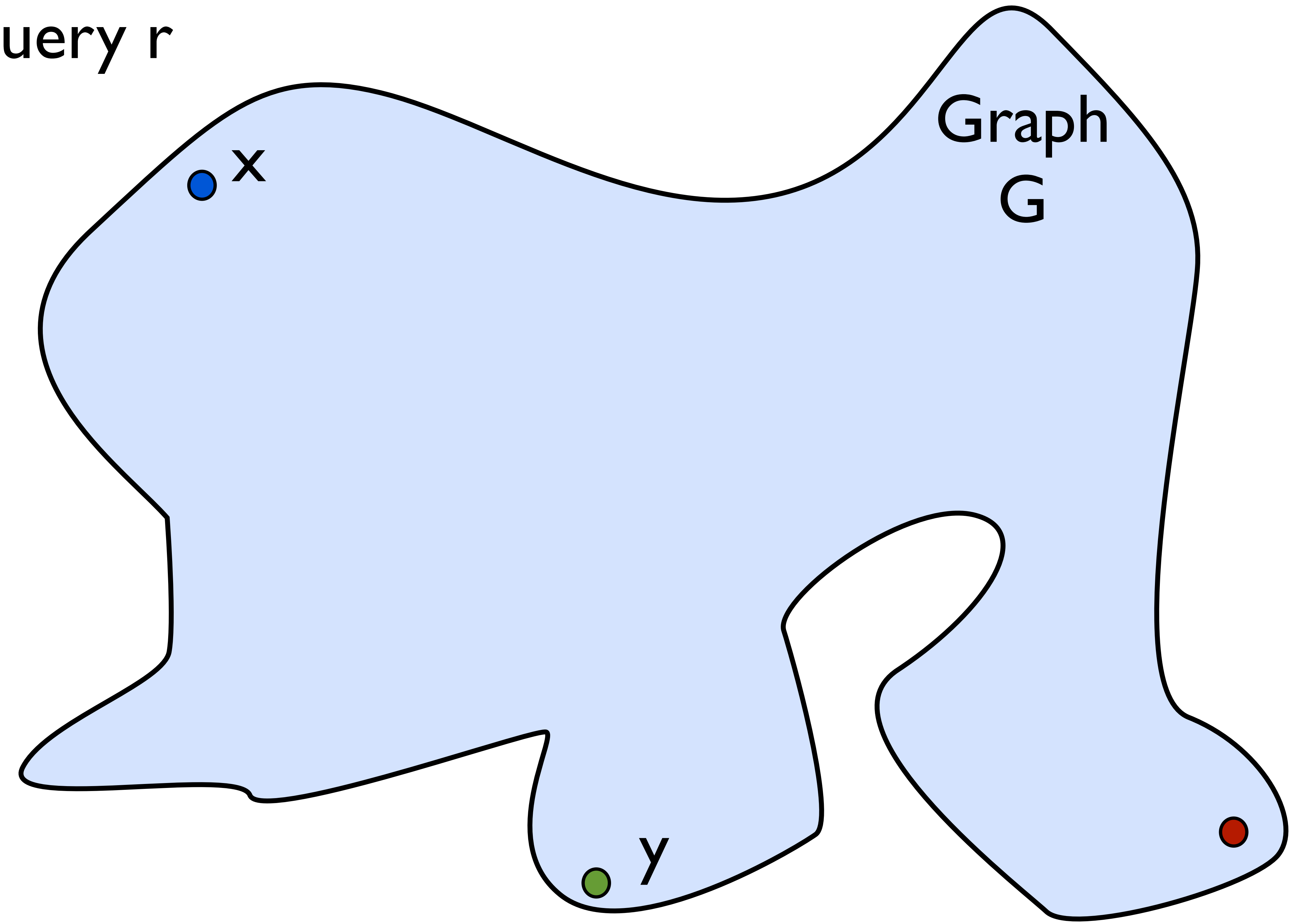
instead of
(,)



More concrete

Reg Path Query r

Selects
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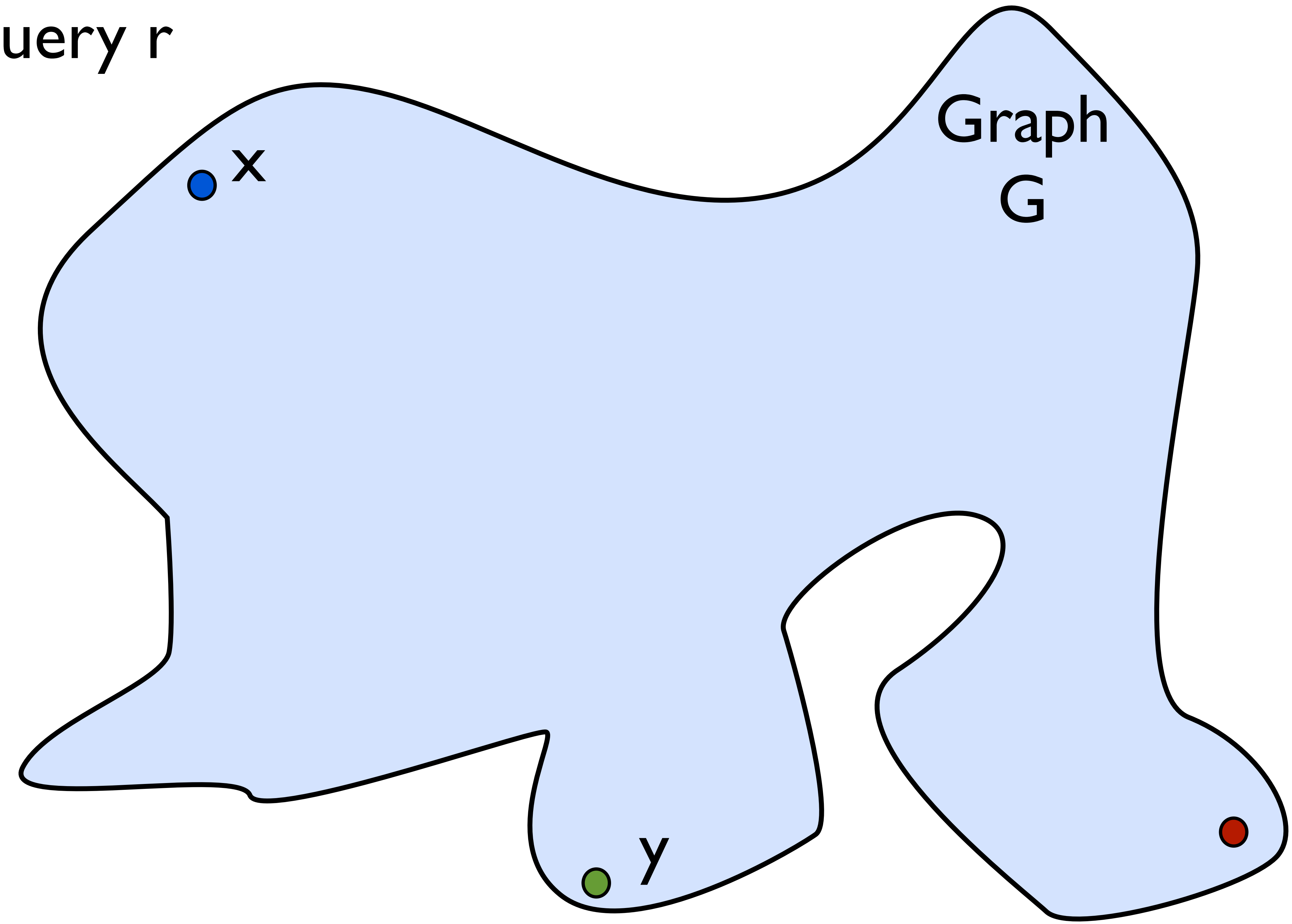
Why?



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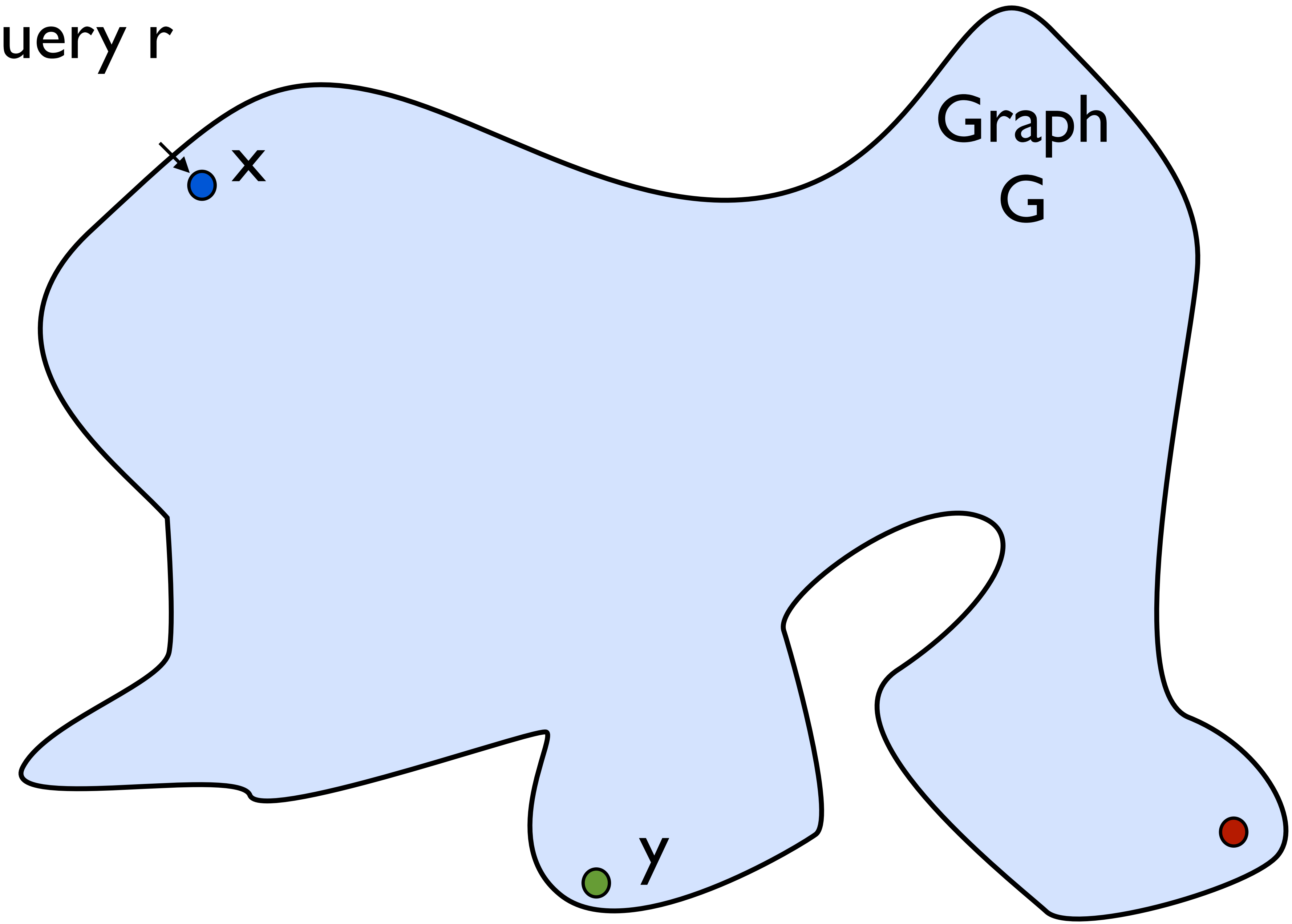

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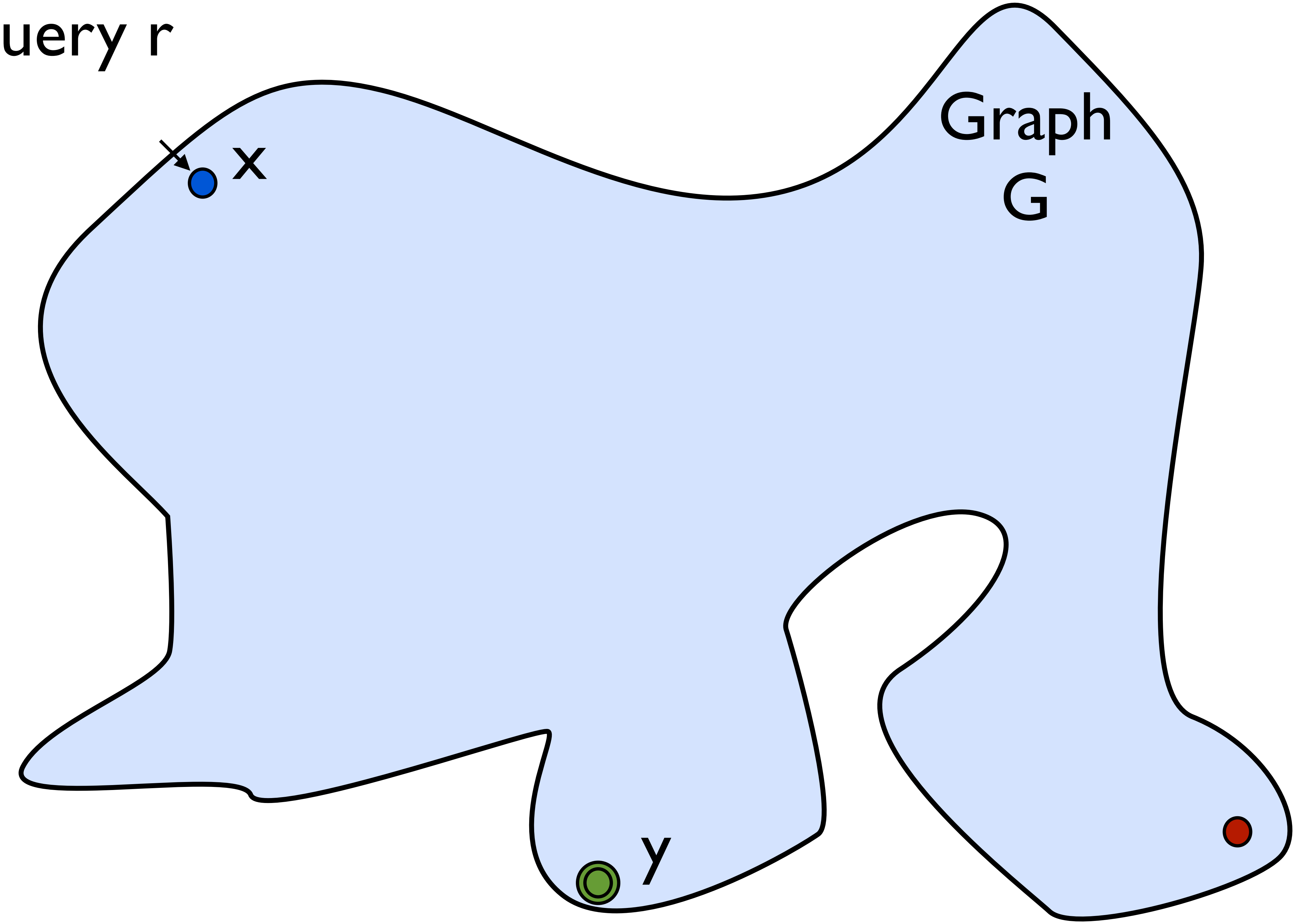

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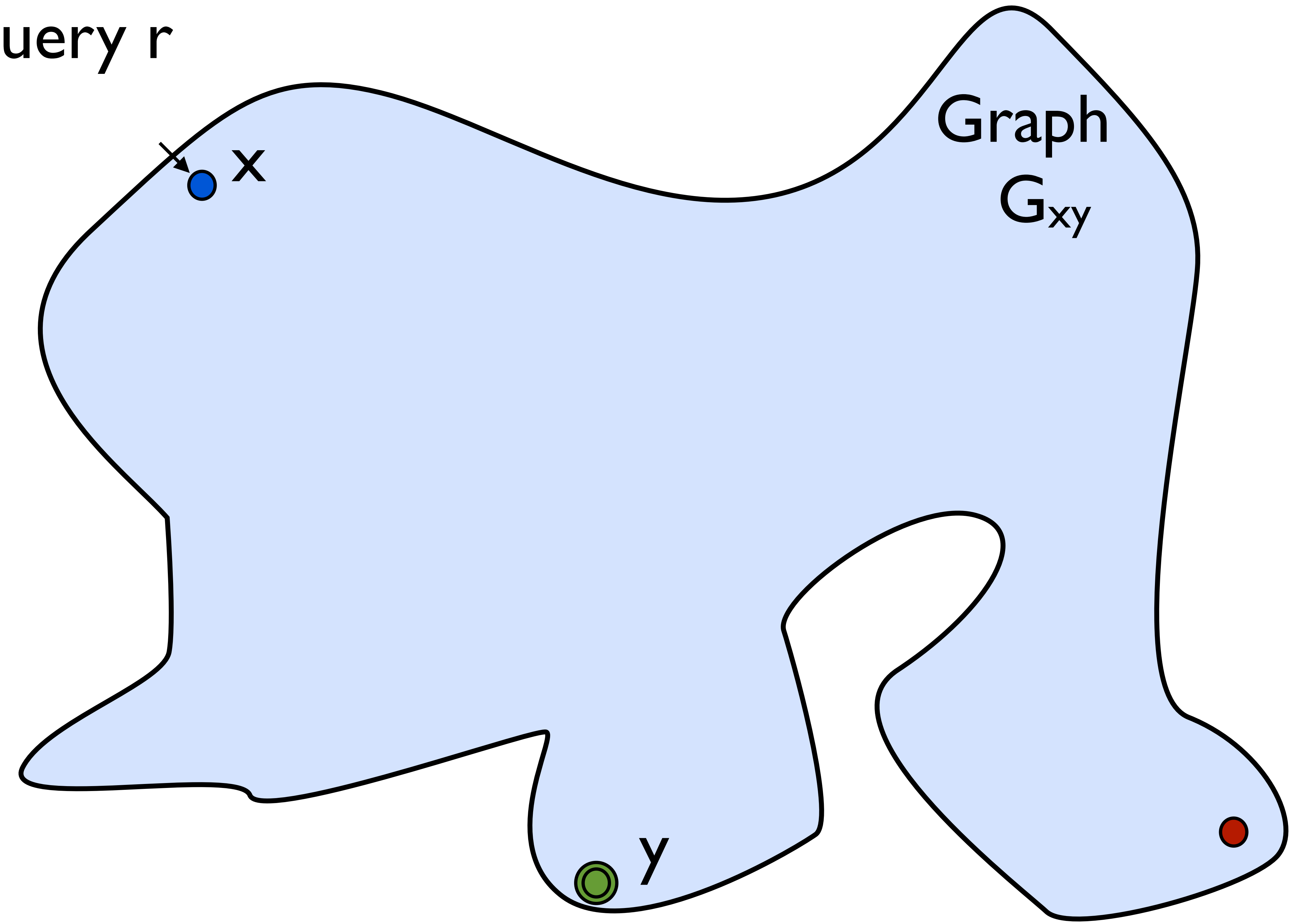

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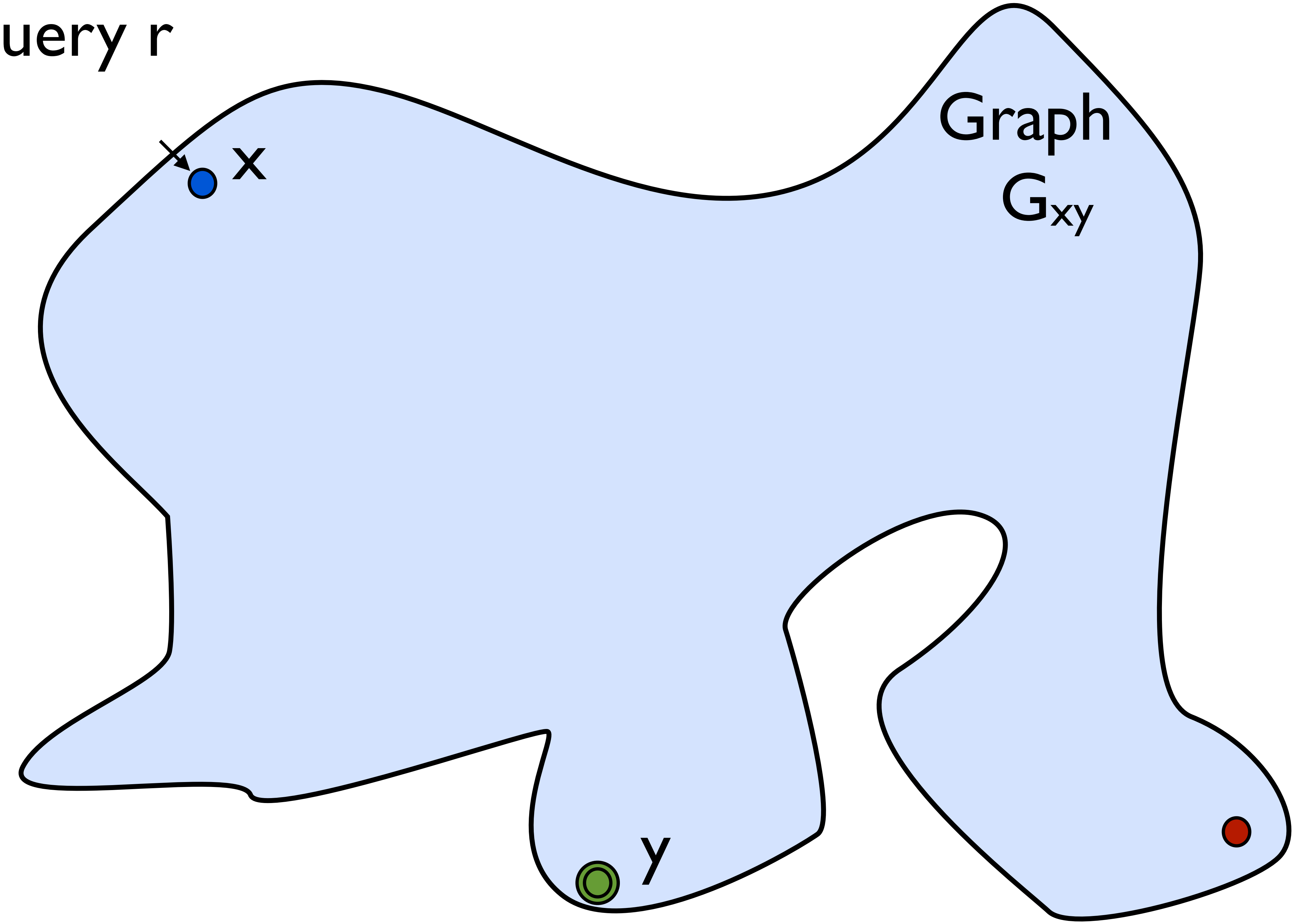

Why?

Because $L(r)$ and $L(G_{xy})$ have empty intersection

This problem boils down to

Given

- regular word language I
- regular word language E

why is I disjoint from E?

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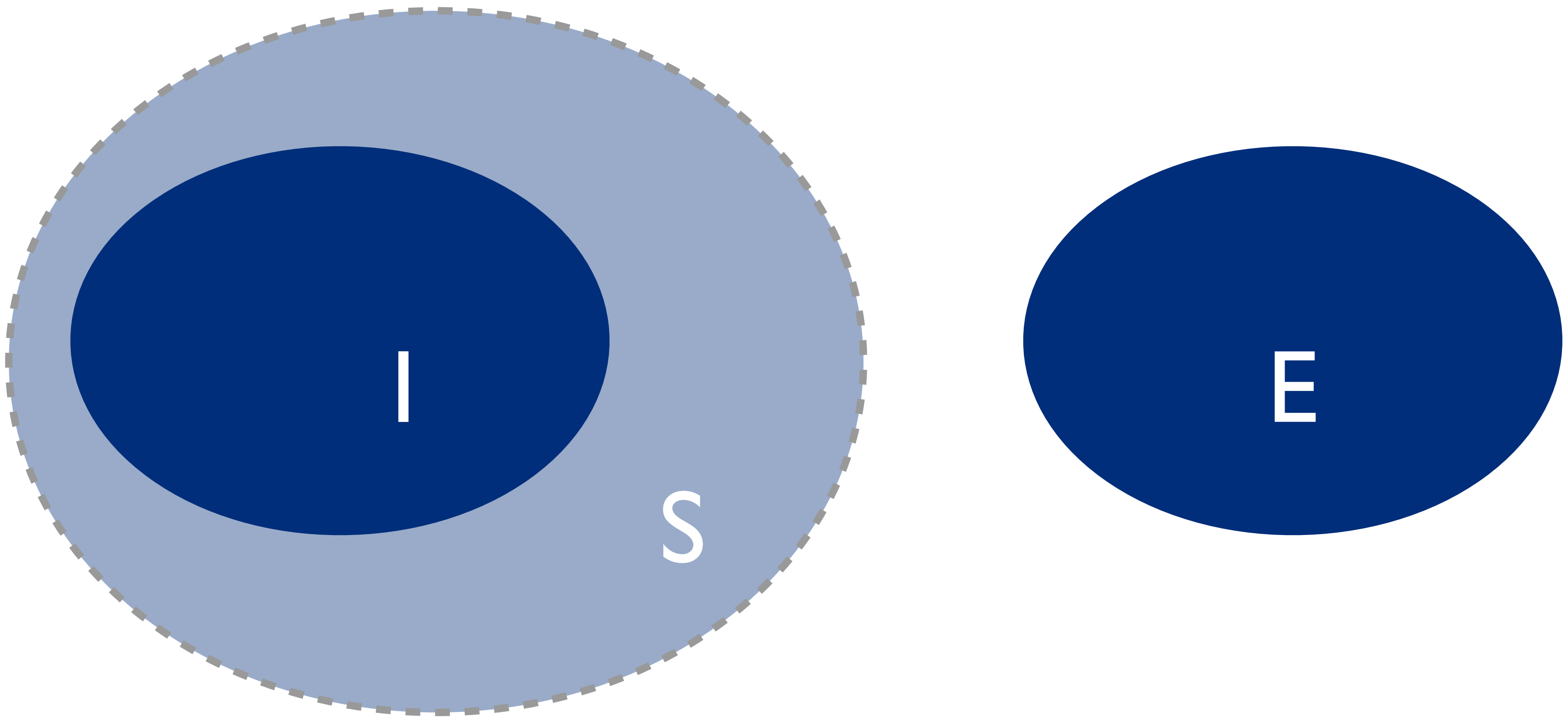

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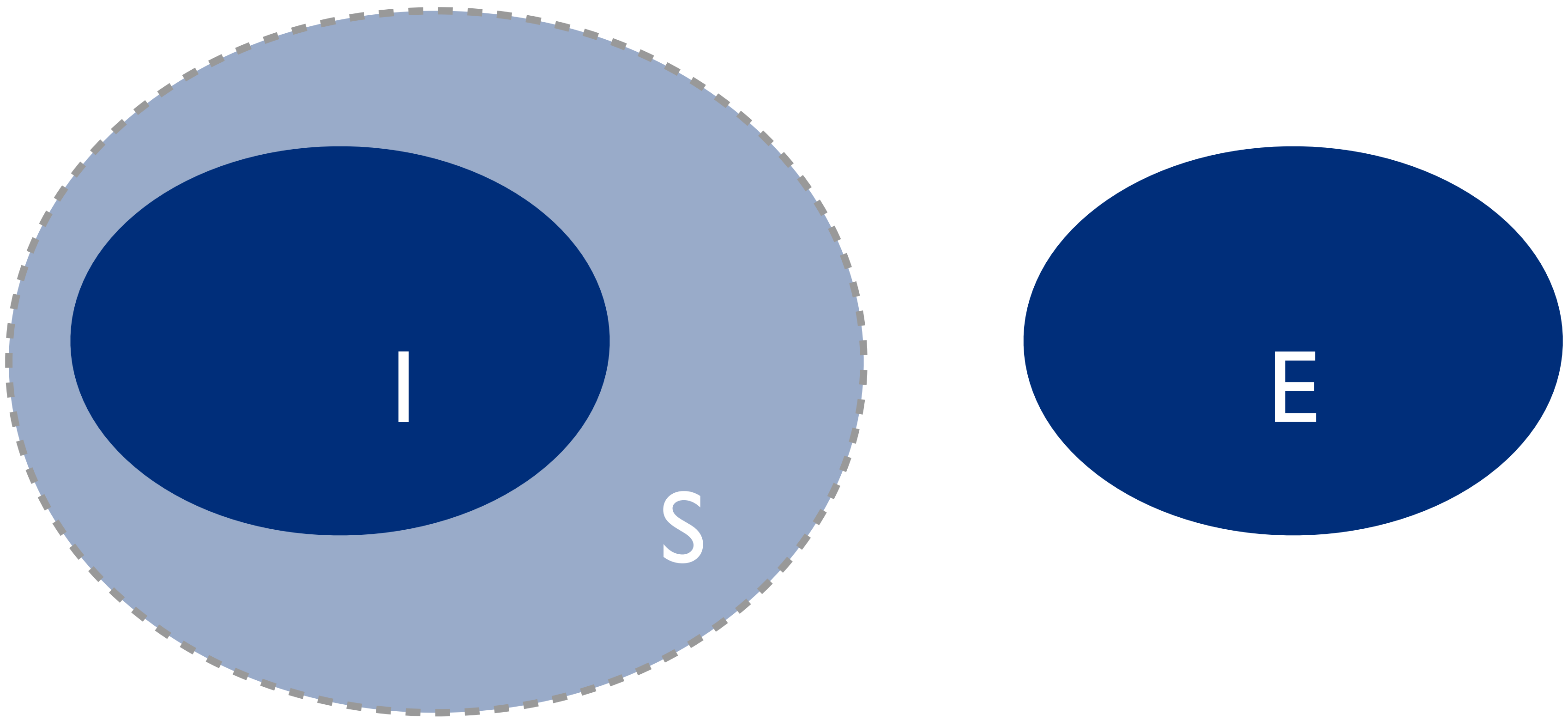
Separation



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I and E are *separable by* family F
if some S from F separates them

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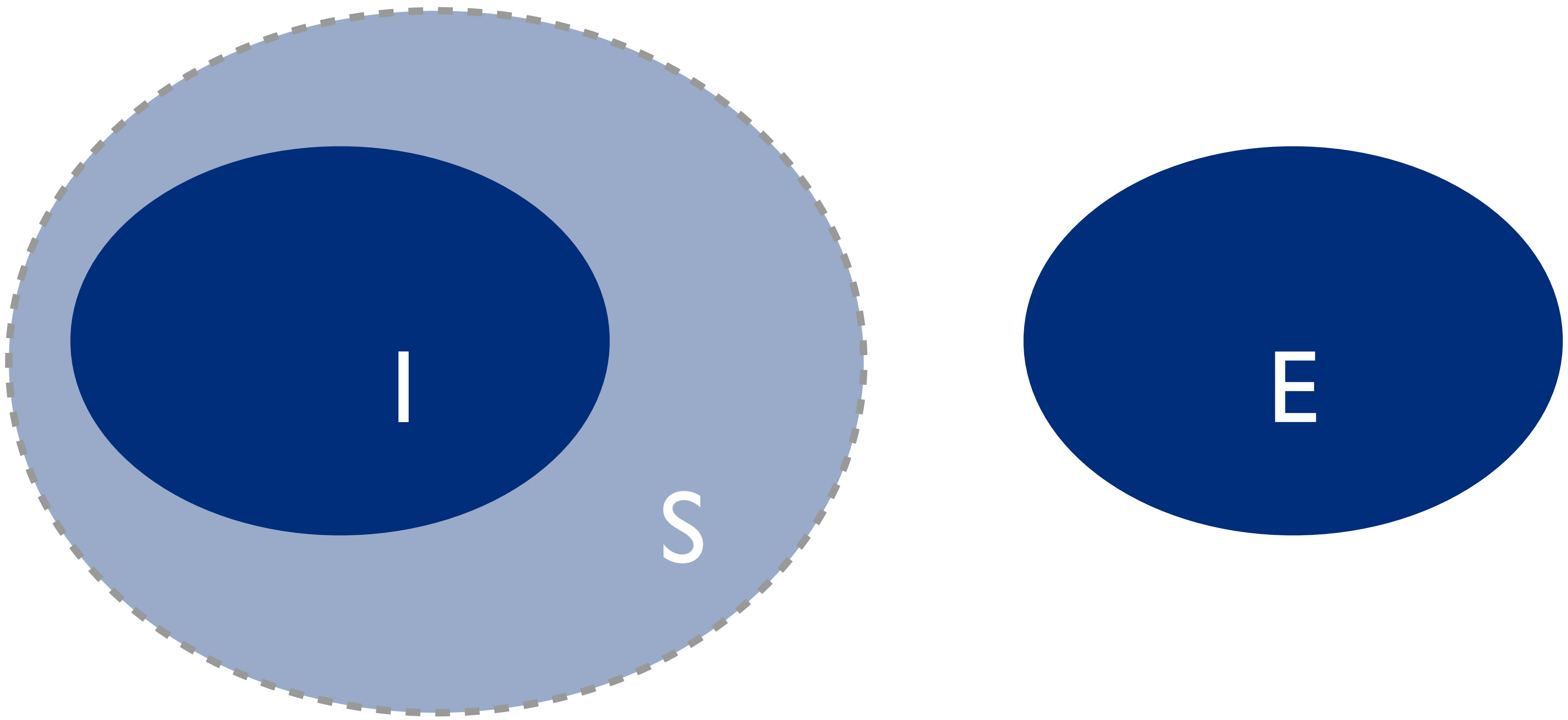
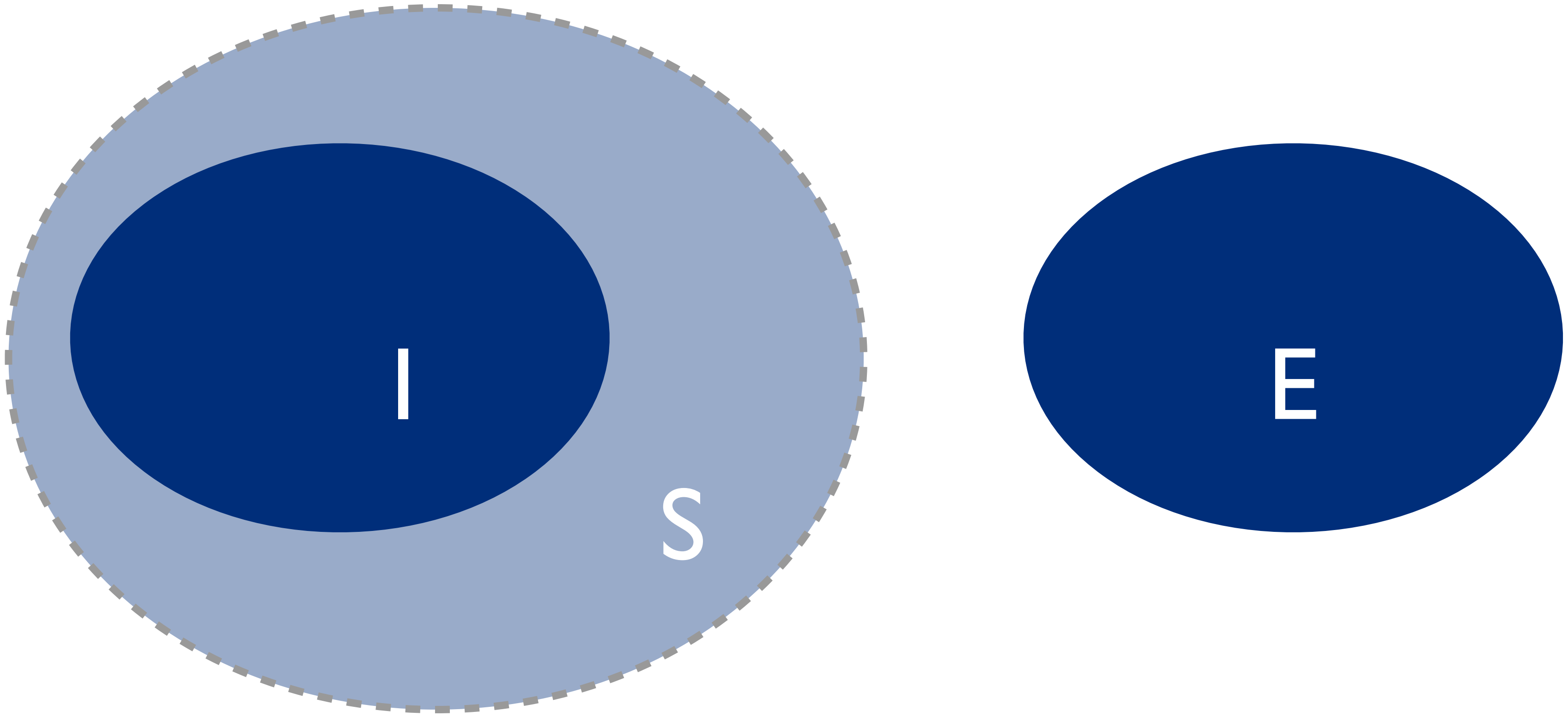


S *separates* I from E

Which $\mathcal{F}$?

I and E are *separable by* family $\mathcal{F}$
if some S from $\mathcal{F}$ separates them

Separation



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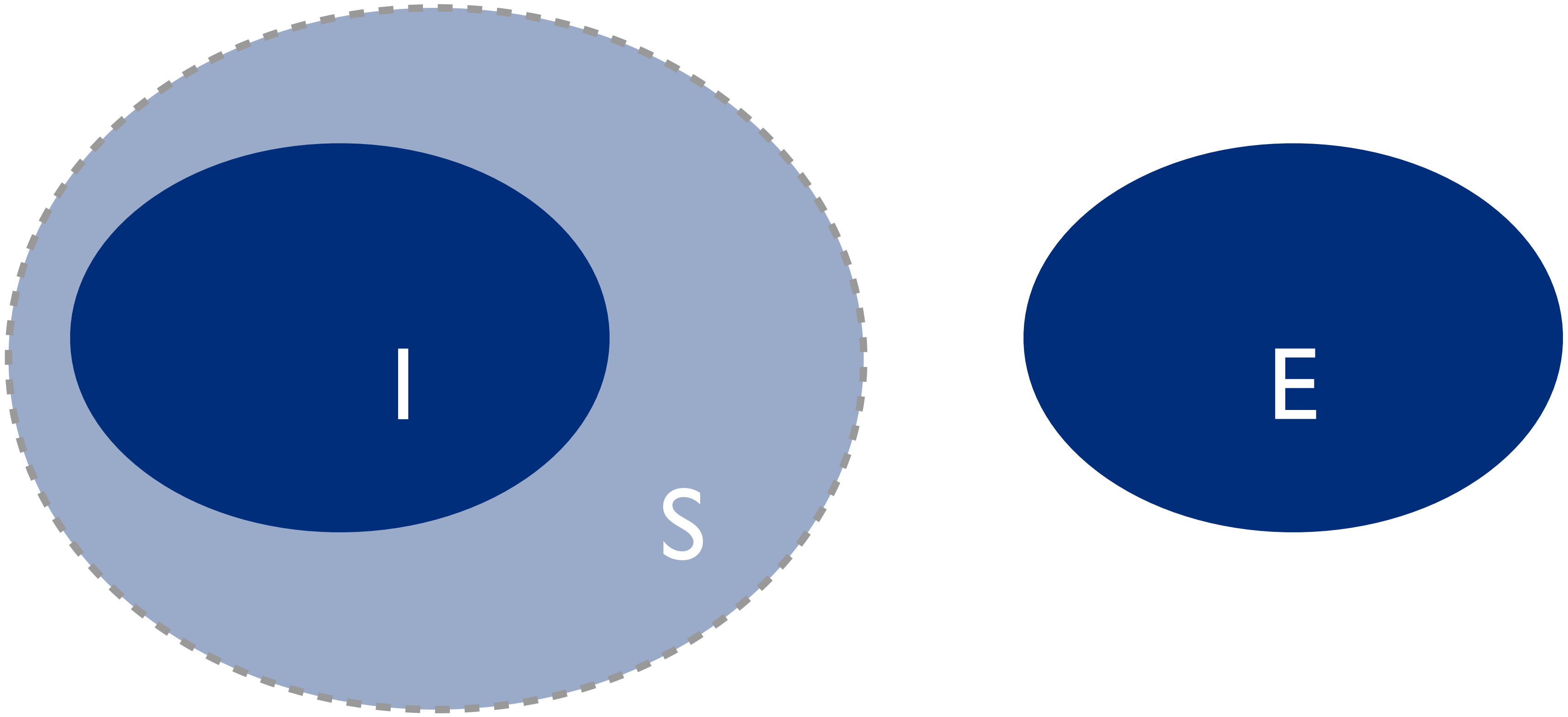


Here: S will come from families of

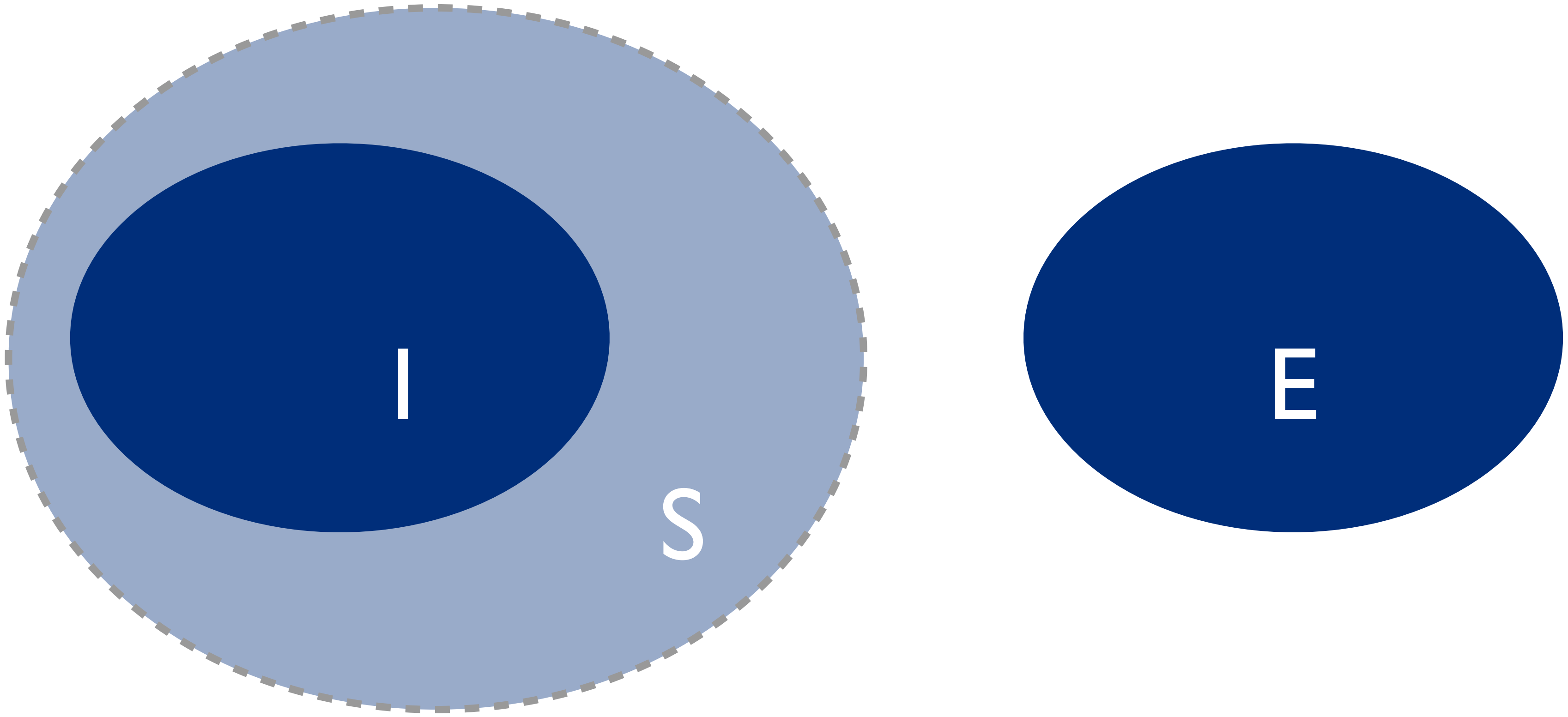
subword languages

...abc...

abc...

...abc

Separation



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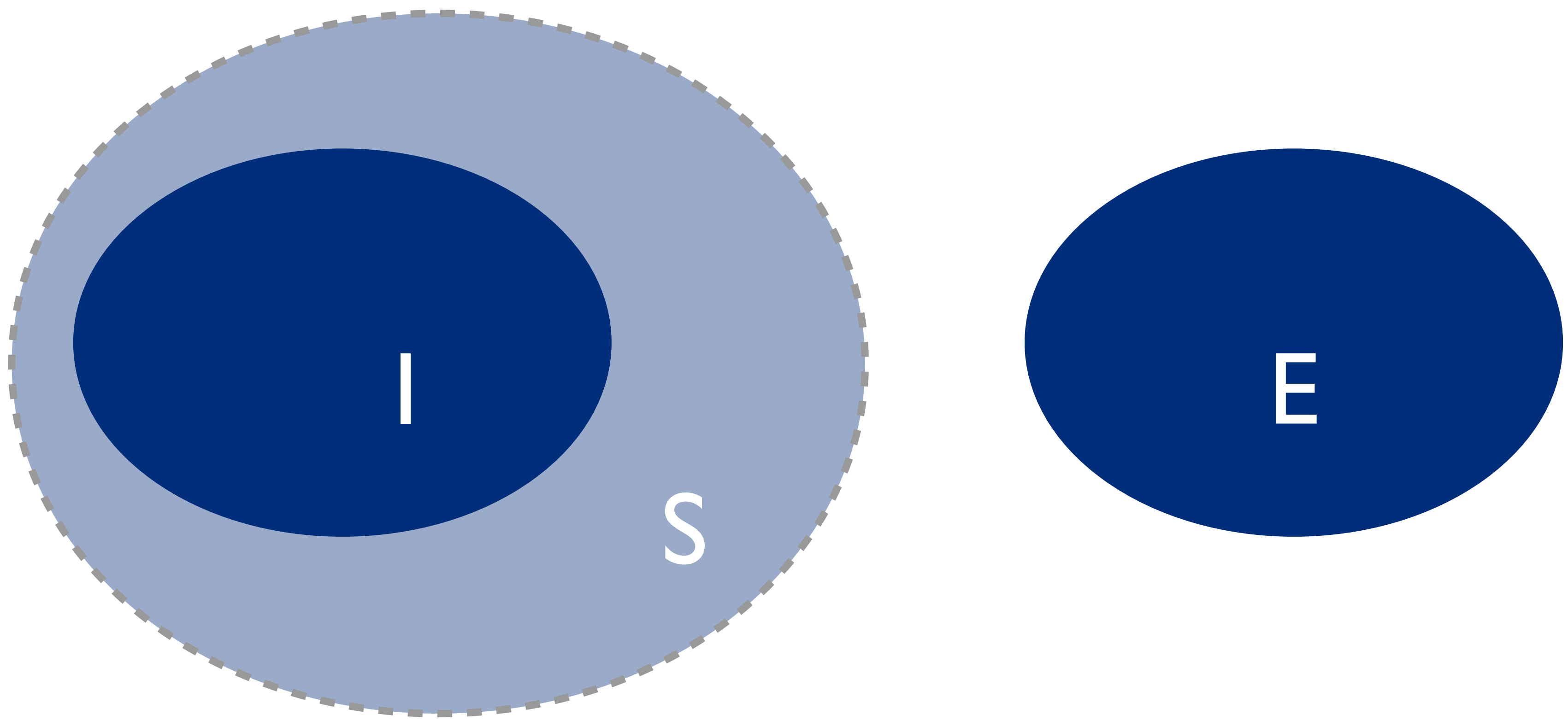

Here: S will come from families of

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subsequence languages

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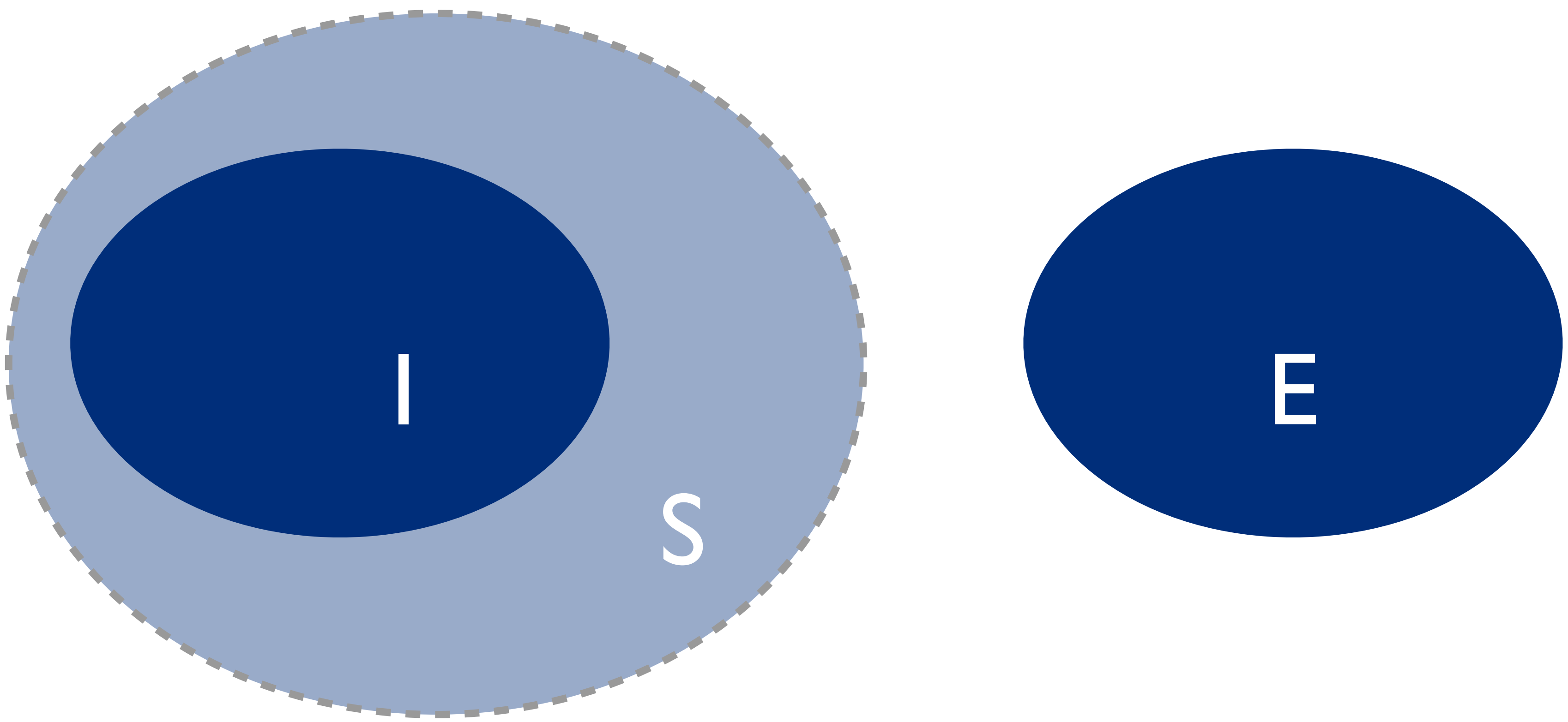
and combinations thereof

Main problem

Separability($\mathcal{F}$)

Given:	Regular languages I and E	(as NFAs)
Question:	Is I separable from E by some S in $\mathcal{F}$?	



So, here, we just decide separability
and our work is still very preliminary

Main problem

Separability($\mathcal{F}$)

Given:	Regular languages I and E	(as NFAs)
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We will now look at different $\mathcal{F}$

Prefixes and Suffixes

A prefix language (over alphabet Σ)

is a language of the form

$$w\Sigma^*$$

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Theorem / Observation:

Separability($\mathcal{F}$) is in PTIME for the following $\mathcal{F}$:

- the prefix languages
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Intuition: "local" explanations are easy to find

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A subsequence language is a language of the form

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Intuition: Non-separability is some kind of reachability

Short Subsequences

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Separability(F) is

- NP-complete for k-subsequence languages
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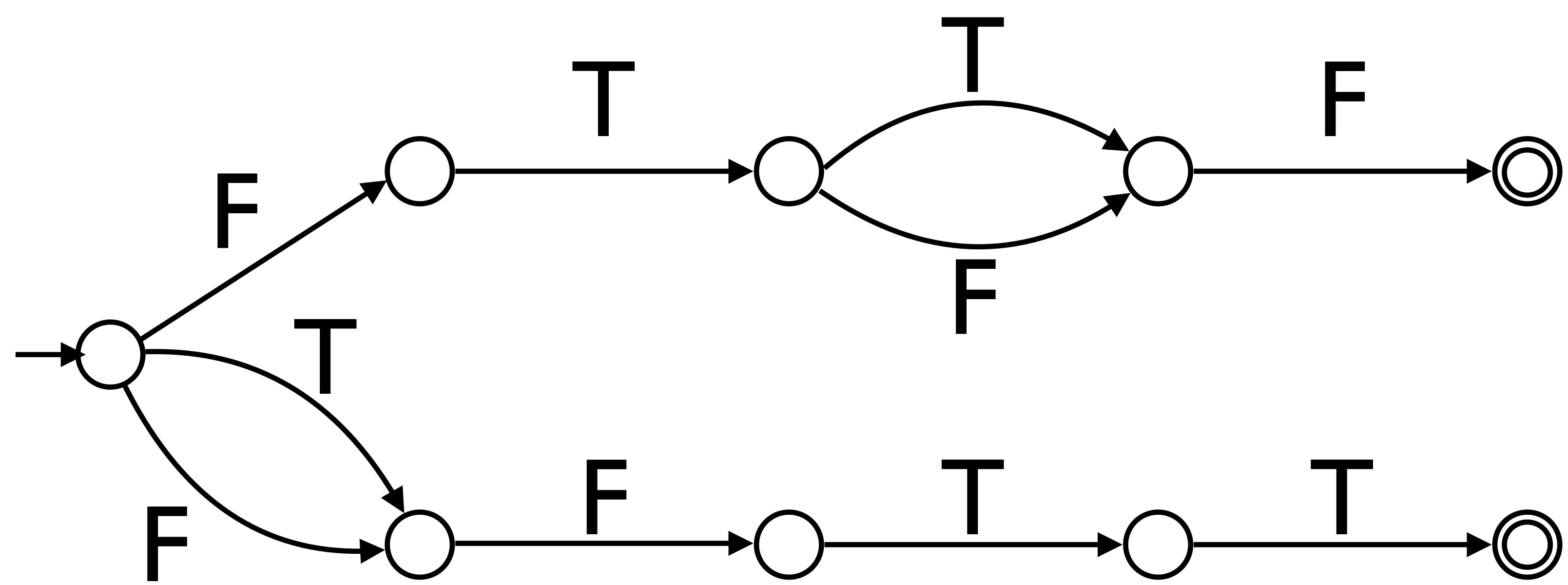

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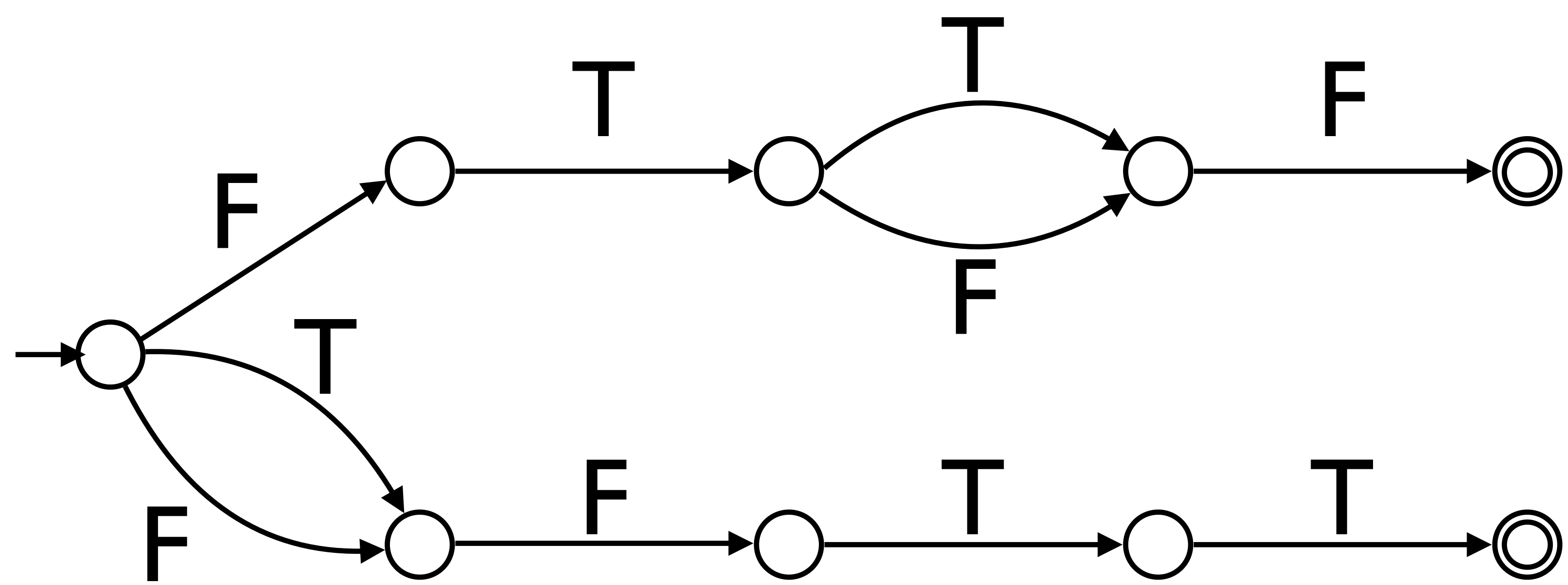

Let I = TFTFTFTF

Short Subsequences

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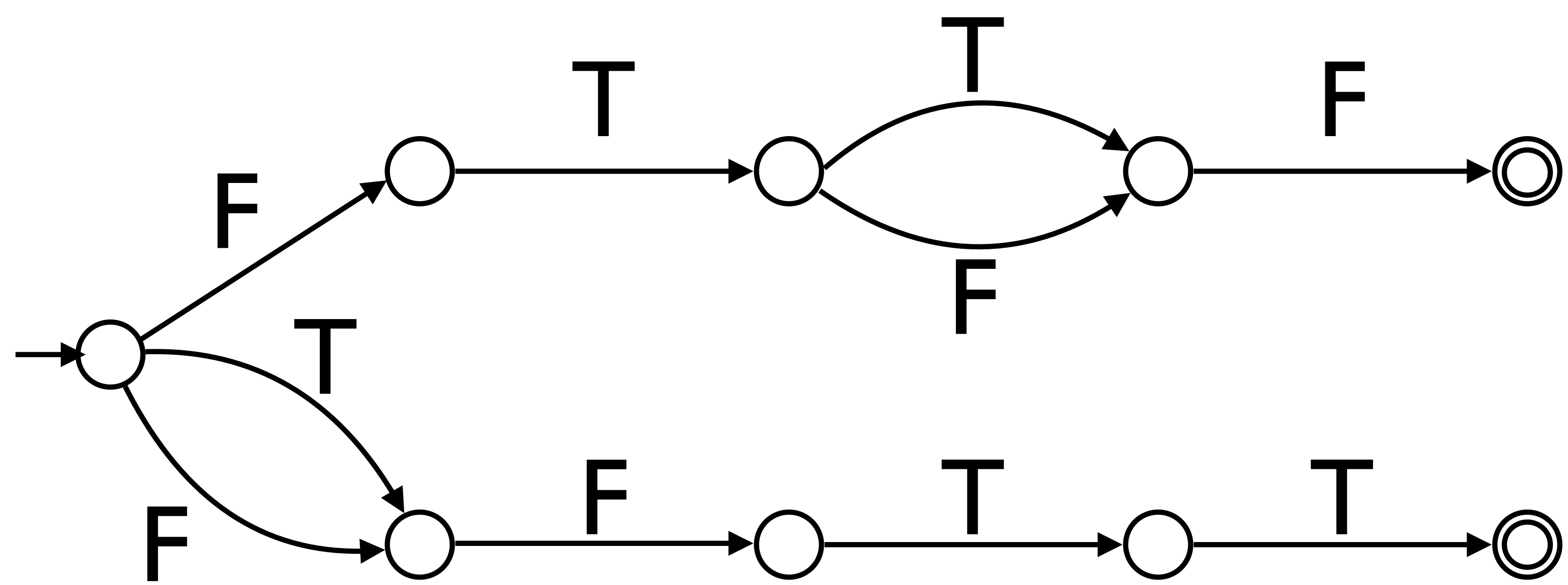
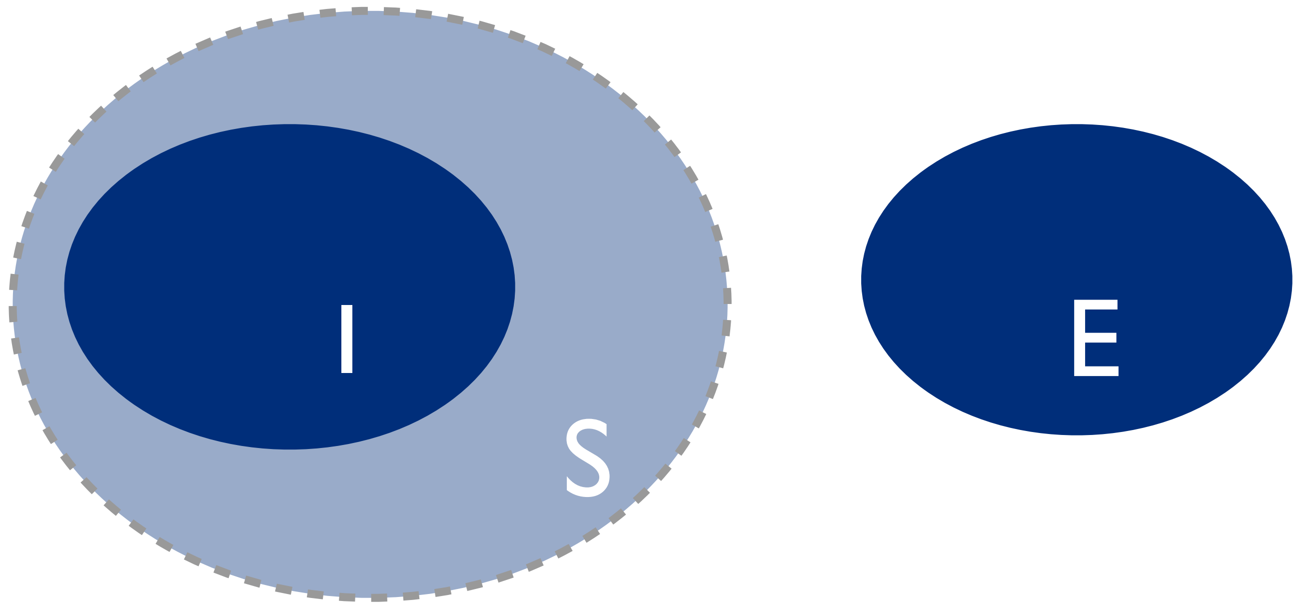
Let E = 

Let I = TFTFTFTF

I is separable from E
by 4-subsequence language
iff
$L(E) \neq (T+F)(T+F)(T+F)(T+F)$

Subsequences: Restricting I and E

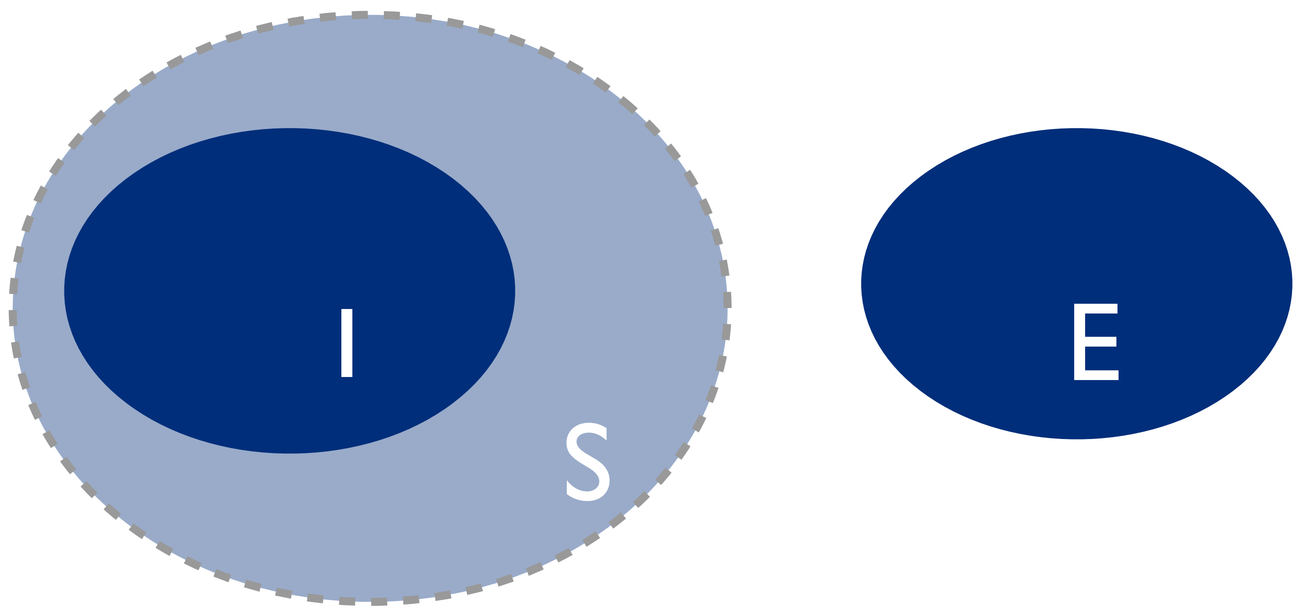


What happens if we restrict I or E?

If E has a constant-size core-approximation, then separability of I from E is in PTIME for

- k-subsequence languages and
- unions / intersections / positive combinations of k-subsequence languages

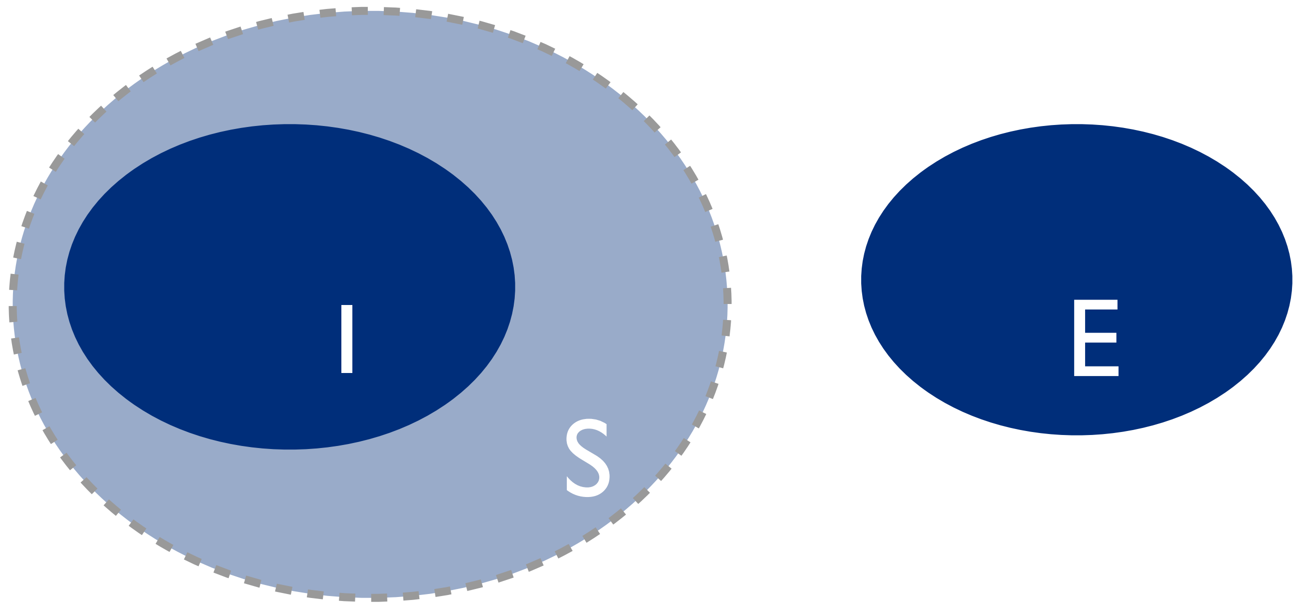
Subsequences: Restricting I and E



Core-approximation of an NFA:

- Collapse all strongly connected components
- Perform bisimulation minimization

Subsequences: Restricting I and E



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- k-subsequence languages

This technique can be extended to show tractable separability by k-subsequences of constant-length words

$\ldots a_1 b_1 \ldots a_2 b_2 \ldots \quad . \; . \; . \quad \ldots a_k b_k \ldots$

Recap and Other Results

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A promising case seems to be
k-subsequences of constant-length subwords

Concluding Remarks

Separation is a very general and exciting problem:

Why is language 1 disjoint from language 2?

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Why is language 1 disjoint from language 2?

It's been a research topic in language theory for a while now but seems to be gaining momentum nowadays

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Interesting related question: Why is a result in the answer?

Thank you!

Questions?