Regular Expressions with Counting: Weak versus Strong Determinism

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Introduction

Regular Expressions

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- for standard regular expressions r and s,
  rs, $r+s$, $r?$, and r^* are in REG(Σ)

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Counting is used in, e.g.,...

- XML Schema
- egrep
- Perl patterns

Outline

Deterministic Regular Expressions

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Deterministic expressions are used in, e.g., ...

- Document Type Definitions (DTD)
- SGML
- XML Schema

Strong and Weak Determinism

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- a^* is strongly deterministic and equivalent

Notation

- $\mathrm{DET}_S(\Sigma)$: strongly deterministic $\mathrm{REG}(\Sigma)$
- $\mathrm{DET}_W(\Sigma)$: weakly deterministic $\mathrm{REG}(\Sigma)$

Weak / Strong Determinism with Counting

Weak: "When matching a string from left to right,
it's always clear which position in the expression to match next"

Example

- $(b?a^{2,3})^{3,3}b$ is not weakly deterministic (witness: $aaaaaab\ldots$)
- $(b?a^{2,3})^{2,2}b$ is weakly deterministic

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Strong: "It should also be clear which operator to use next"

Example

- $(a^{1,2})^{3,4}$ is weakly deterministic, but not strongly deterministic
- $(a^{2,2})^{3,4}$ is weakly and strongly deterministic

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Do we want weak or strong determinism?

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What you need to know about REG(Σ)

Theorem (Implicit in Brüggmann-Klein, 1993; Brügg.-Klein,Wood, 1998)

Expressive power in a picture:

$$DET_S(\Sigma) = DET_W(\Sigma) \subsetneq REG(\Sigma)$$

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Theorem (Implicit in Brüggmann-Klein, 1993)

Every weakly deterministic expression can be made strongly deterministic in linear time

What you need to know

Brüggemann-Klein and Wood, 1998

Testing weak determinism of an expression is easy ($\mathcal{O}(n^2)$)

Core operation: Glushkov(r)

Consider $r = c(a+b)^*a$

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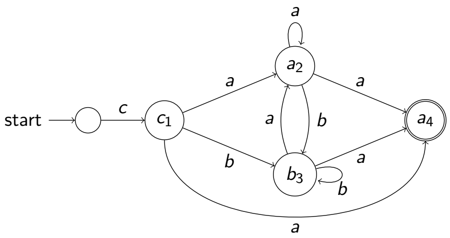

Expression r is deterministic iff Glushkov(r) is a DFA

Complexity of (Weakly) Deterministic Expressions

- **MEMBERSHIP**: Given string w and expression r, is $w \in L(r)$?
- **INCLUSION**: Given expressions r_1, r_2, is $L(r_1) \subseteq L(r_2)$?
- **INTERSECTION**: Given expressions $r_1, \ldots, r_n$, is $\bigcap_i L(r_i) \neq \emptyset$?

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Theorem

For (weakly) deterministic expressions:

- **MEMBERSHIP** *is in* $\mathcal{O}(n^2)$
- **INCLUSION***: in* **PTIME** *[Stearns, Hunt 1981]*
- **INTERSECTION***:* **PSPACE***-complete*

[Mar., Neven, Schwentick 2004]

Questions

The Situation for deterministic expressions...

	Without counting	
Expressiveness	$\text{DET}_S(\Sigma) = \text{DET}_W(\Sigma) \subsetneq \text{REG}(\Sigma)$	
Succinctness	$\text{DET}_S(\Sigma) \approx \text{DET}_W(\Sigma)$	
Det-Test	easy (Glushkov)	
$\exists$-Det-Test	**EXPTIME**	
Membership	$\mathcal{O}(n^2)$	
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The equalities...

- $\mathrm{DET}_W(\Sigma) = \mathrm{DET}_S^{\#}(\Sigma)$ $(|\Sigma| > 1)$
- $\mathrm{DET}_W(\Sigma) = \mathrm{DET}_W^{\#}(\Sigma)$ $(|\Sigma| = 1)$

are non-trivial!

Witness separating languages:

- $(a^{2,3}b?)^*$ is in $\mathrm{DET}_W^{\#}(\Sigma)$, but not in $\mathrm{DET}_W(\Sigma)$
- $(aaa)^*(a + aa)$ is in $\mathrm{REG}(\Sigma)$, but not in $\mathrm{DET}_W(\Sigma)$

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$r = (a^{2^n+1, 2^{n+1}})^{1,2}$
"all strings of as of length $2^n + 1$ until 2^{n+2}, but not of length $2^{n+1} + 1$"

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Corollary

The above theorem holds for unary languages

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Counter Automata

Counter Automaton: $(Q, q_0, C, \delta, F, \tau)$

Ingredients:

- Q: states, q_0: initial state

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Guard(C): Boolean combination of true, false, $c = k$, $c < k$
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Remark

These are very similar to [McQueen]

Counter Automata: Determinism

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(q, α), where q is a state, and $\alpha : C \to \mathbb{N}$

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Note

Testing determinism is **PSPACE**-complete

From Expressions to Automata

We extend the Glushkov construction to $\mathrm{REG}^{\#}(\Sigma)$
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Theorem

For every expression $r \in REG^{\#}(\Sigma)$,

- $L(r) = L(Glushkov^{\#}(r))$, *and*
- *r is strongly deterministic $\Leftrightarrow$ $Glushkov^{\#}(r)$ is deterministic*

Outline

Testing Determinism

Theorem

- *Testing weak determinism for* $REG^{\#}(\Sigma)$ *is in time* $\mathcal{O}(n^3)$
  (Kilpeläinen and Tuhkanen 2007)

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Theorem

Testing strong determinism for $REG^{\#}(\Sigma)$ *is in time* $\mathcal{O}(n^3)$

Inclusion, Intersection

What should we expect? To put you in the right mood

Theorem (Gelade, Mar., Neven 2007)

- **INCLUSION** *for* $REG^{\#}(\Sigma)$ *is* **EXPSPACE**-*complete*
- **INTERSECTION** *for* $REG^{\#}(\Sigma)$ *is* **PSPACE**-*complete*

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Theorem

- **INCLUSION** *for* $DET^{\#}_{S}(\Sigma)$ *is in* **PSPACE** *(from automata)*
- **INTERSECTION** *for* $DET^{\#}_{S}(\Sigma)$ *and* $DET^{\#}_{W}(\Sigma)$ *is* **PSPACE**-*complete*
- **MEMBERSHIP** *for* $DET^{\#}_{S}(\Sigma)$ *is in* $\mathcal{O}(n^3)$ *(from automata)*

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Concluding Remarks

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Succinctness	$\text{DET}_S(\Sigma) \approx \text{DET}_W(\Sigma)$	strong $>_{\text{exp}}$ weak
Det-Test	easy (Glushkov)	easy (+ Glushkov$^{\#}$)
$\exists$-Det-Test	**EXPTIME**	2**EXPTIME** (strong)
Membership	$\mathcal{O}(n^2)$	$\mathcal{O}(n^3)$
Complexity	**PTIME**/ **PSPACE**	**PSPACE** (strong)

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Do we want weak or strong determinism?

Thank you for listening