XML Research for Formal Language Theorists

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TU Dortmund

Goal of this talk

XML Research vs Formal Languages

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XML Research vs Formal Languages

- XML benefits from Formal Language Theory
  - XML schemas $\approx$ tree automata
  - XPath patterns $\approx$ regular expressions
  - Formal Language Theory has a nice algorithmic toolbox

Goal of this talk

XML Research vs Formal Languages

- XML benefits from Formal Language Theory
  - XML schemas ≈ tree automata
  - XPath patterns ≈ regular expressions
  - Formal Language Theory has a nice algorithmic toolbox
- Formal Language Theory benefits from XML
  - XML motivates interesting Formal Language problems

Warning

- Rather informal strongly biased survey

Outline

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1. Introduction to XML

2. An FLT Approach to XML Research
   - Document Type Definitions
   - XML Queries
   - Extended Document Type Definitions and XML Schema
   - Characterizations of single-type EDTDs

3. From XML to Formal Language Theory
   - Complexity of Regular Expressions
   - Constructions on Regular Expressions
   - Automata Minimization

Searching the Internet

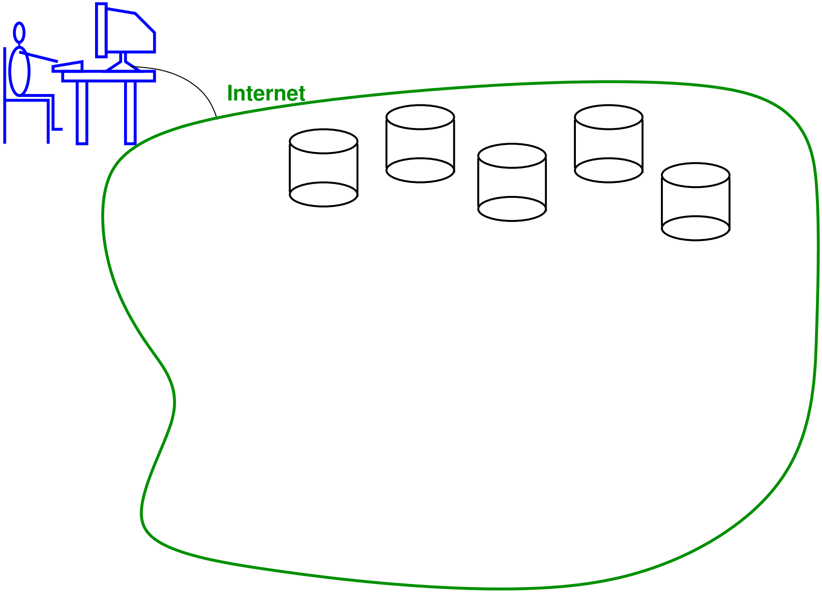
Enough with these sissy keyword searches!

Searching the Internet

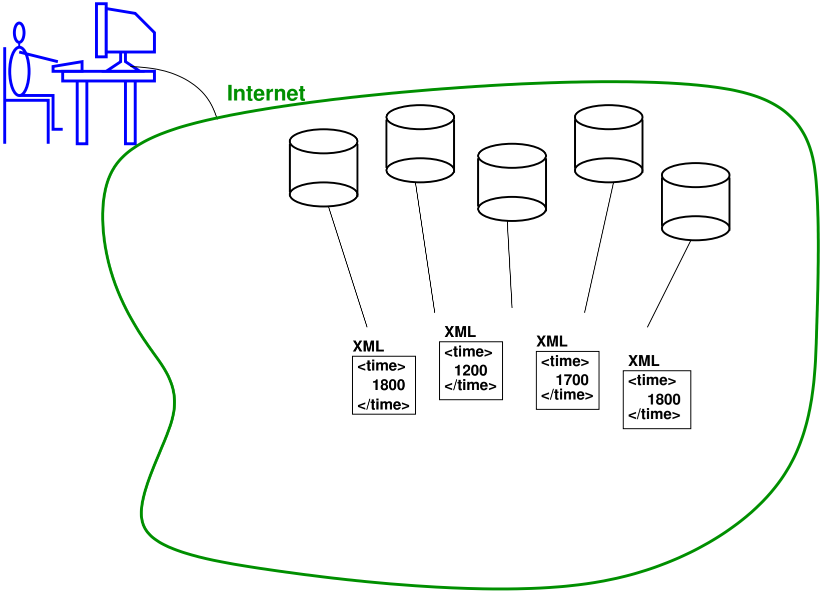
A *real* search

Where can I buy a flatscreen-TV, in a store at most 20km from Dresden, that is open tomorrow until 18:00?

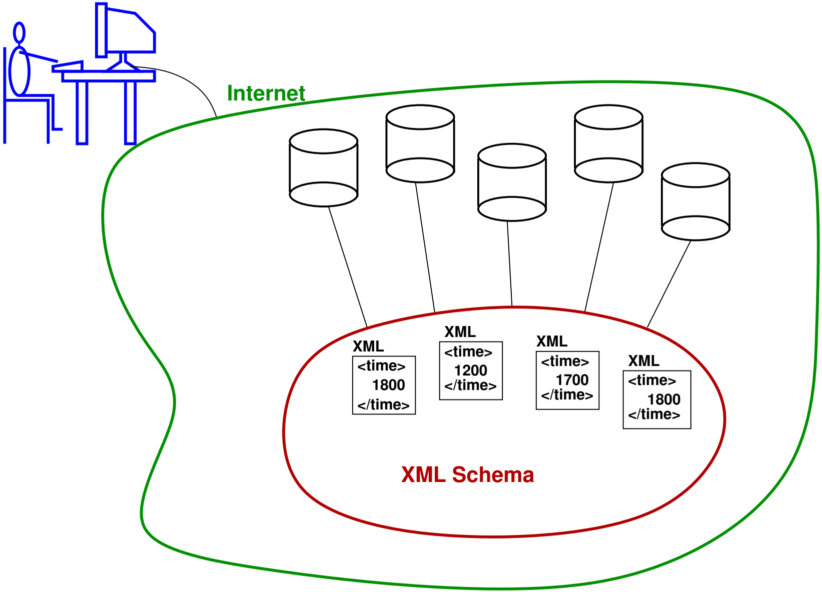
An Example



An Example


Internet
XML
<time> 1800 </time>
XML
<time> 1200 </time>
XML
<time> 1700 </time>
XML
<time> 1800 </time>

An Example



Internet
XML
<time>
1800
</time>
XML
<time>
1200
</time>
XML
<time>
1700
</time>
XML
<time>
1800
</time>
XML Schema

A self-describing data format

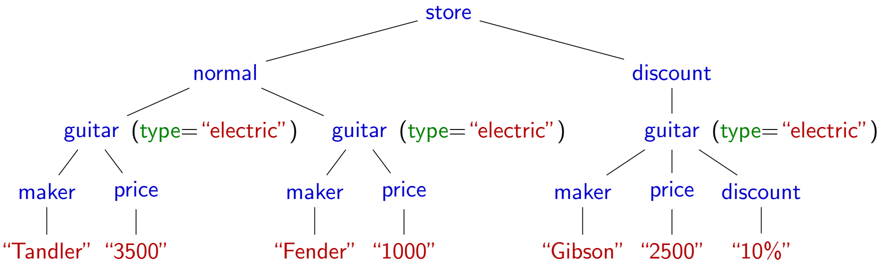
```
<store>
    <normal>
        <guitar type="electric">
            <maker> Tandler </maker>
            <price> 3500 </price>
        </guitar>
        <guitar type="electric">
            <maker> Fender </maker>
            <price> 1000 </price>
        </guitar>
    </normal>
    <discount>
        <guitar type="electric">
            <maker> Gibson </maker>
            <price> 2500 </price>
            <discount> 10% </discount>
        </guitar>
    </discount>
</store>
```

element: `<title>...</title>`

start tag: `<title>`
end tag: `</title>`

XML as a hierarchical structure

Example



Abstraction: ordered, unranked, labeled tree (with data-values)

XML schema languages

Schema

A schema defines the set of allowable labels and the way they can be structured.

Advantages

- automatic validation
- automatic integration of data
- automatic translation
- query optimization
- provides a user with a concrete semantics of the document
- aids in the specification of meaningful queries over XML data

XML schema languages

In formal language theoretic terms

A schema defines a tree language.

Example

- DTDs (W3C) — CFGs with REs
- XML Schema (W3C) — $\not\approx$ tree automata
- Relax NG (Clark, Murata) — $\approx$ tree automata
- several dozen others (DSD, Schematron, ...)

Summary slide

What to remember?

- XML is an international standard for data exchange
- XML documents or XML data are simply ordered unranked labeled trees with data values
- a schema defines a tree language (no data values — in this talk)

Outline

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Document Type Definitions (DTDs)

Example

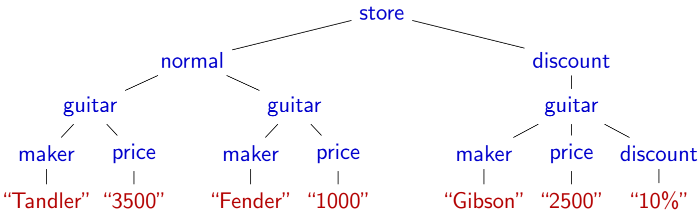
```
<!DOCTYPE store [
    <!ELEMENT store     (normal,discount)>
    <!ELEMENT normal    (guitar*)>
    <!ELEMENT discount  (guitar+)>
    <!ELEMENT guitar    (maker,price,discount?)>
    <!ELEMENT maker     (#PCDATA)>
    <!ELEMENT price     (#PCDATA)>
    <!ELEMENT discount  (#PCDATA)>
]>
```

Corresponding grammar (start symbol store)

$$
\begin{array}{lcl}
\text{store} & \longrightarrow & \text{normal discount} \\
\text{normal} & \longrightarrow & \text{guitar}^* \\
\text{discount} & \longrightarrow & \text{guitar}^+ \\
\text{guitar} & \longrightarrow & \text{maker price discount?} \\
\text{maker} & \longrightarrow & \text{DATA} \\
\text{price} & \longrightarrow & \text{DATA} \\
\text{discount} & \longrightarrow & \text{DATA}
\end{array}
$$

Document Type Definitions (DTDs)

XML Document



Corresponding grammar (start symbol store)

store	$\rightarrow$	normal discount
normal	$\rightarrow$	guitar*
discount	$\rightarrow$	guitar$^+$
guitar	$\rightarrow$	maker price discount?
maker	$\rightarrow$	DATA
price	$\rightarrow$	DATA
discount	$\rightarrow$	DATA

Extended Context-free grammars as a formal abstraction

Definition

A DTD is a triple (Σ, d, s_d) where

- Σ is a finite alphabet
- $s_d \in \Sigma$ is the start symbol
- $d : \Sigma \to \mathrm{RE}(\Sigma)$ maps every Σ-symbol to a regular expression over Σ

Definition

A tree t satisfies d (is valid) iff

- the root of t is labeled s_d
- for every node v labeled a the string formed by the children of v belongs to $d(a)$.

Optimization questions: from FLT to XML

Schema containment ($\subseteq$)

Given: Schemas d_1, d_2
Question: Is $L(d_1) \subseteq L(d_2)$?

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DTD containment reduces to containment of regular expressions

$$d_1 \subseteq d_2 \qquad \text{iff} \qquad d_1(a) \subseteq d_2(a),\ \forall a \in \Sigma$$

(when d_1 and d_2 are reduced).

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Theorem (Meyer, Stockmeyer, 1973)

Containment of regular expressions is **PSPACE***-complete.*

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Corollary

DTD containment is **PSPACE**-complete.

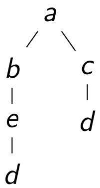
Outline

Queries for XML

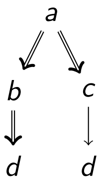
Conjunctive Queries over Trees

XPath

Tree:



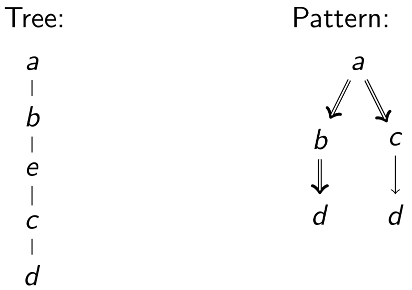
Pattern:



Pattern Matching

- Tree matches Pattern if there is a homomorphism h : Pattern $\longrightarrow$ Tree
- Homomorphism doesn't have to be injective

XPath



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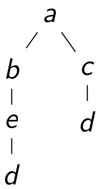

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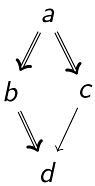

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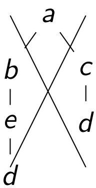

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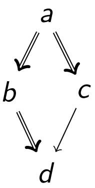

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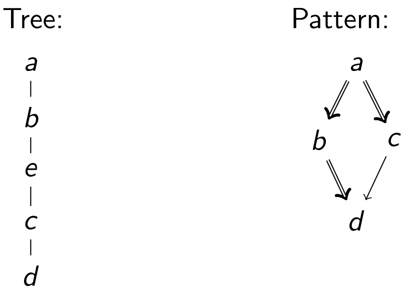

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Query Optimization

$L(Q)$: the set of trees that match query Q

Query Containment

Given two queries Q_1 and Q_2, is $L(Q_1) \subseteq L(Q_2)$?

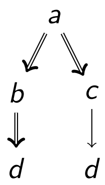
Query Containment w.r.t. a DTD

Given Q_1, Q_2, and a DTD d, is $L(Q_1) \cap L(d) \subseteq L(Q_2)$?

XPath Query Optimization

Formal Language Theory to the Rescue!

XPath Query



Lemma

For each XPath query Q there is an Alternating Tree Automaton A s.t.

$$L(Q) = L(A)$$

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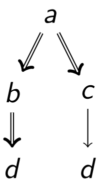

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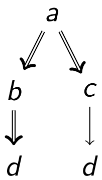
Moreover, $|A|$ *is polynomial in* $|Q|$

XPath Query



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Theorem

- *XPath Containment is in* **EXPTIME**
- *XPath Containment w.r.t. DTDs is in* **EXPTIME**

XPath Query Optimization

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Theorem

- *XPath Containment (tree pattern fragment) is* **NP***-complete [Miklau, Suciu 2002]*
- *XPath Containment (with $\neg$ and $\vee$) is* **EXPTIME***-complete [Marx 2004]*
- *XPath Containment w.r.t. DTDs is* **EXPTIME***-complete [Neven, Schwentick 2003]*

Conjunctive Query Optimization

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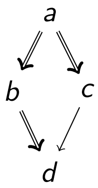
Conjunctive Query



Lemma (Björklund, Mar., Schwentick 2008)

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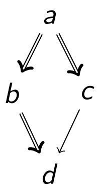
Conjunctive Query Optimization

Formal Language Theory to the Rescue!

Conjunctive Query



Lemma (Björklund, Mar., Schwentick 2008)

For each Conjunctive Query Q there is an Alternating Tree Automaton A s.t.

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But, $|A|$ is exponential in $|Q|$

Conjunctive Query Optimization

Formal Language Theory to the Rescue!

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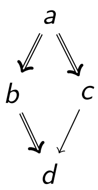

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Conjunctive Query Optimization

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$$L(Q) = L(A)$$

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Theorem

- *CQ Containment w.r.t. DTDs is* **2EXPTIME**-*complete*

  [Björklund, Mar., Schwentick 2008]

Conjunctive Query Optimization

Formal Language Theory to the Rescue!

Lemma (Björklund, Mar., Schwentick 2008)

For each Conjunctive Query Q there is an Alternating Tree Automaton A s.t.

$$L(Q) = L(A)$$

But, $|A|$ is exponential in $|Q|$ and this is optimal

Theorem

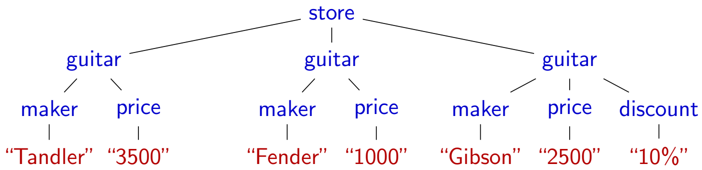
- *CQ Containment is Π_2^P-complete [Björklund, Mar., Schwentick 2007]*
- *CQ Containment w.r.t. DTDs is* **2EXPTIME***-complete [Björklund, Mar., Schwentick 2008]*

Outline

Extended DTDs

Grammar based approach to unranked regular tree languages

tree t



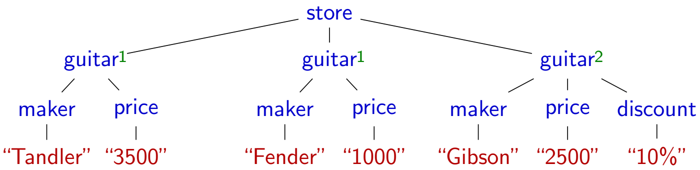
Example

$$
\begin{array}{lcl}
\text{store} & \to & (\text{guitar}^1)^* \, (\text{guitar}^2)^+ \\
\text{guitar}^1 & \to & \text{maker price} \\
\text{guitar}^2 & \to & \text{maker price discount}
\end{array}
$$

Extended DTDs

Grammar based approach to unranked regular tree languages

Typed tree t'



Example

$$\begin{array}{lcl} \text{store} & \rightarrow & (\text{guitar}^1)^* \ (\text{guitar}^2)^+ \\ \text{guitar}^1 & \rightarrow & \text{maker price} \\ \text{guitar}^2 & \rightarrow & \text{maker price discount} \end{array}$$

Extended DTDs

Grammar based approach to unranked regular tree languages

Definition (Papakonstantinou, Vianu, 2000)

Let $\Sigma^{\mathbb{N}} := \{\sigma^n \mid \sigma \in \Sigma, n \in \mathbb{N}\}$ be the alphabet of types.

An extended DTD (EDTD) is a tuple $D = (\Sigma, d, s_d)$, where (d, s_d) is a (finite) DTD over $\Sigma \cup \Sigma^{\mathbb{N}}$.

A tree t is valid w.r.t. D if there is an assignment of types such that the typed tree is a derivation tree of d.

Example

$$
\begin{array}{lcl}
\text{store} & \rightarrow & (\text{guitar}^1)^* \ (\text{guitar}^2)^+ \\
\text{guitar}^1 & \rightarrow & \text{maker price} \\
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EDTDs versus Tree Automata

Theorem (Papakonstantinou, Vianu, 2000, BMW)

Non-deterministic (unranked) tree automata and EDTDs define precisely the class of (homogeneous) regular unranked tree languages.

EDTDs versus Tree Automata

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Example

EDTD

$$\begin{array}{lcl} \text{store} & \rightarrow & (\text{guitar}^1)^* \ (\text{guitar}^2)^+ \\ \text{guitar}^1 & \rightarrow & \text{maker price} \\ \text{guitar}^2 & \rightarrow & \text{maker price discount} \end{array}$$

NTA

$$\begin{array}{lcl} \delta(\mathit{store}, \mathit{store}) & = & (\text{guitar}^1)^* \ (\text{guitar}^2)^+ \\ \delta(\mathit{guitar}^1, \mathit{guitar}) & = & \text{maker price} \\ \delta(\mathit{guitar}^2, \mathit{guitar}) & = & \text{maker price discount} \end{array}$$

Does XML Schema correspond to EDTDs?

```
<xs:element name="store">
  <xs:complexType>
    <xs:sequence>
      <xs:element name="guitar" type="1"
                                minOccurs="0"
                                maxOccurs="unbounded"/>
      <xs:element name="guitar" type="2"
                                minOccurs="1"
                                maxOccurs="unbounded"/>
    </xs:sequence>
  </xs:complexType>
</xs:element>
```

Does XML Schema correspond to EDTDs?

```
<xs:element name="store">
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    <xs:sequence>
      <xs:element name="guitar" type="1"
                                minOccurs="0"
```

Rejected by XML Schema validator

Violates the Element Declarations Consistent Constraint.

```
                                minOccurs="1"
                                maxOccurs="unbounded"/>
    </xs:sequence>
  </xs:complexType>
</xs:element>
```

A formalization of XML Schema: single-type EDTDs

XML Schema 1: Element Declarations Consistent constraint (Section 3.8.6)

It is illegal to have two elements of the same name [. . .] but different types in a content model [. . .].

A formalization of XML Schema: single-type EDTDs

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Definition (Murata, Lee, Mani, 2001)

A single-type EDTD is an EDTD for which in no regular expression two types b^i and b^j with $i \neq j$ occur.

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Not single-type

$$\begin{array}{lcl} \text{store} & \to & (\text{guitar}^1)^* \; (\text{guitar}^2)^+ \\ \text{guitar}^1 & \to & \text{maker price} \\ \text{guitar}^2 & \to & \text{maker price discount} \end{array}$$

A formalization of XML Schema: single-type EDTDs

Definition (Murata, Lee, Mani, 2001)

A single-type EDTD is an EDTD in which in no regular expression two types b^i and b^j with $i \neq j$ occur.

Example

store	$\longrightarrow$	normal discount
normal	$\longrightarrow$	$(\text{guitar}^1)^*$
discount	$\longrightarrow$	$(\text{guitar}^2)^+$
guitar^1	$\longrightarrow$	maker price
guitar^2	$\longrightarrow$	maker price discount

A formalization of XML Schema: single-type EDTDs

Formal abstraction

XML Schema $\approx$ single-type EDTDs

A formalization of XML Schema: single-type EDTDs

Formal abstraction

XML Schema $\approx$ single-type EDTDs

Immediate Questions

- What kind of languages can be defined by single-type EDTDs?
- Is it decidable whether an EDTD rewritten to an equivalent single-type EDTD?

smart XML Schema validator

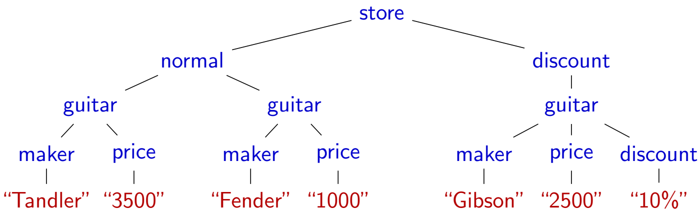
Outline

Properties of single-type EDTDs

Three properties

1. Single-type EDTDs admit unique top-down typing
2. Closure under a certain form of subtree exchange
3. Characterization as a pattern-based language

(1) Single-type EDTDs: simple top-down typing



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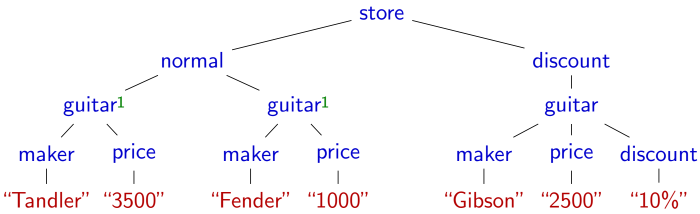

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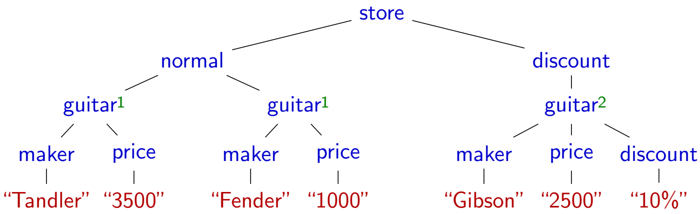

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guitar^1	$\rightarrow$	maker price
guitar^2	$\rightarrow$	maker price discount

(1) Single-type EDTDs: simple top-down typing

Algorithm to validate and type a tree (Murata et al., 2001)

Given: tree t and single-type EDTD $D = (\Sigma, d, a^0)$

1. Check if root of t is labeled with a, assign type a^0
2. for every interior node u with type b^i, test whether the children of u match $\mu(d(b^i))$. If so, assign unique type to every child. Else fail.

$\mu(a^1 + b^1 c^2) = a + bc$

(1) Single-type EDTDs: simple top-down typing

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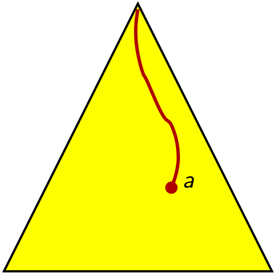
Corollary

Single-typedness implies unique top-down typing.

(2) An exchange property of single-type EDTDs

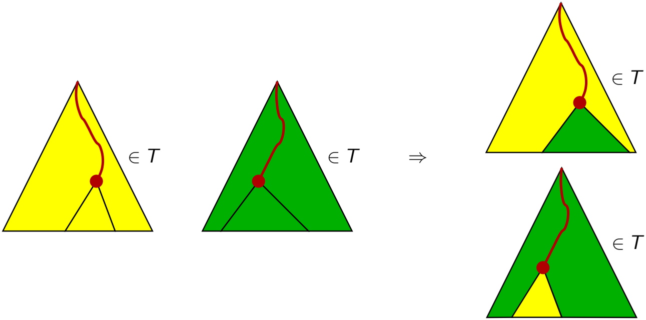
The Ancestor-String



(2) An exchange property for single-type EDTDs

Ancestor-Guarded Subtree Exchange

T is a regular tree language

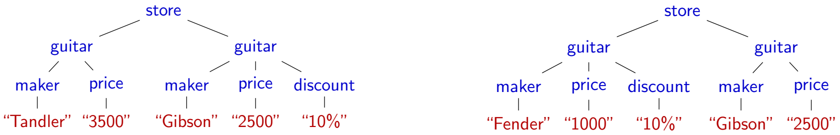


Theorem (Mar., Neven, Schwentick 2005)

A regular tree language is definable by a single-type EDTD iff it is closed under ancestor-guarded subtree exchange.

(2) Tool for proving inexpressibility

"At least one discount guitar" is not single-type



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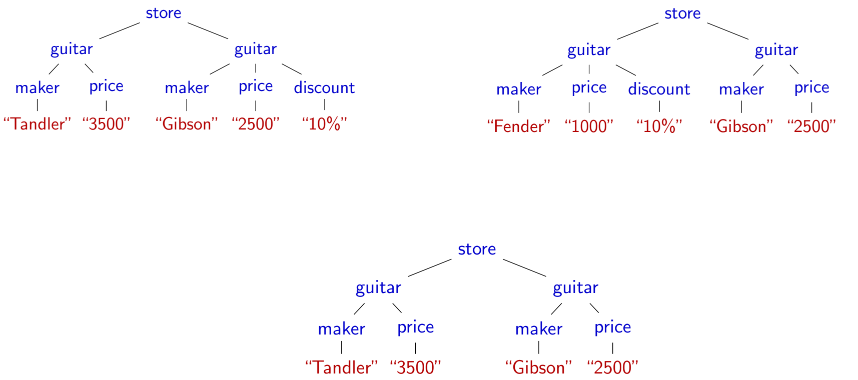

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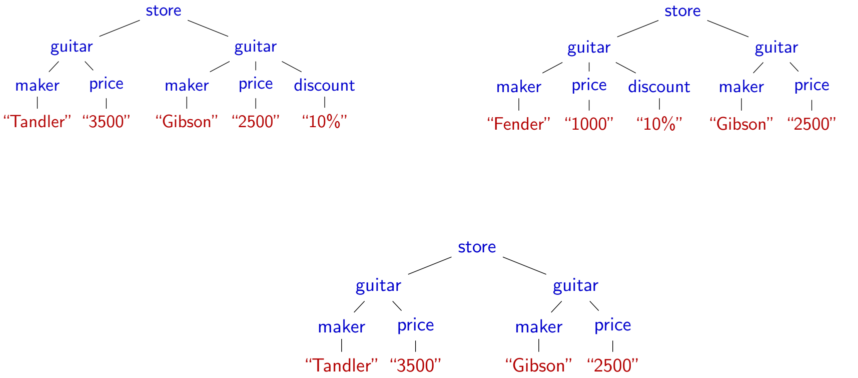


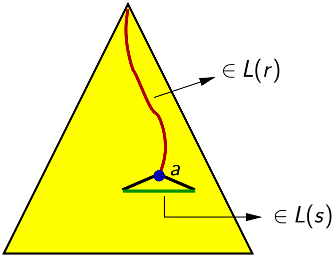
Single-type EDTDs are not closed under union or complement.

(3) Pattern-based Language

Making dependencies explicit

Definition

An ancestor-based DTD A is a set of rules $r \longrightarrow s$ where r and s are regular expressions over Σ.



Definition

A tree t is valid w.r.t. A iff for every vertex v there is some $r \longrightarrow s$ such that v's ancestor string matches r and the children of v match s.

(3) Pattern-based Language

Making dependencies explicit

single-type EDTD

$$
\begin{array}{lcl}
\text{store} & \to & \text{normal discount} \\
\text{normal} & \to & (\text{guitar}^1)^* \\
\text{discount} & \to & (\text{guitar}^2)^+ \\
\text{guitar}^1 & \to & \text{maker price} \\
\text{guitar}^2 & \to & \text{maker price discount}
\end{array}
$$

Ancestor-guarded DTD

$$
\begin{array}{lcl}
\text{store} & \to & \text{normal discount} \\
\text{normal} & \to & \text{guitar}^* \\
\text{discount} & \to & \text{guitar}^+ \\
*\cdot \text{normal} \cdot \text{guitar} & \to & \text{maker price} \\
*\cdot \text{discount} \cdot \text{guitar} & \to & \text{maker price discount}
\end{array}
$$

Smart XML Schema validator

Theorem (Mar., Neven, Schwentick, 2005)

Deciding whether an EDTD is equivalent to a single-type EDTD or a DTD is **EXPTIME***-complete.*

Upper bound

Compute single-type closure D' of given EDTD D:
E.g, $a^1 \rightarrow b^1 b^2$, $b^1 \rightarrow c^1$, $b^2 \rightarrow c^2$ becomes

$$a^{\{1\}} \rightarrow b^{\{1,2\}} b^{\{1,2\}}$$
$$b^{\{1,2\}} \rightarrow c^{\{1\}} + c^{\{2\}}$$

$L(D') = L(D)$ iff $L(D)$ is single-type.
We know that $L(D) \subseteq L(D')$.
So, only need to test $L(D') \subseteq L(D)$: $D' \cap \neg D = \emptyset$.

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Summary slide

What to remember?

- XML Schema $\approx$ single-type EDTDs $\subsetneq$ regular tree languages
- single-type EDTDs admit top-down unique typing
- XML Schema can be simply characterized without using types
- Relax NG corresponds to unranked regular tree languages (EDTDs)

Outline

Outline

Complexity of basic decision problems

Theorem (Mar., Neven, Schwentick 2004)

Let R be a class of regular expressions and $\mathscr{C}$ a complexity class. Then the following are equivalent:

- **CONTAINMENT** *for R is in $\mathscr{C}$;*
- **CONTAINMENT** *for $DTD(R)$ is in $\mathscr{C}$;*
- **CONTAINMENT** *for single-type $EDTD(R)$ is in $\mathscr{C}$;*

Theorem (Seidl 1990, 1994)

CONTAINMENT *and* **EQUIVALENCE** *are* **EXPTIME***-complete for EDTDs (even with deterministic REs).*

Complexity of basic decision problems

INTERSECTION: Given a number of schemas $S_1, \ldots, S_n$, decide if $\bigcap_{i=1}^{n} L(S_i) \neq \emptyset$.

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- **INTERSECTION** *for R is in $\mathscr{C}$;*
- **INTERSECTION** *for DTD(R) is in $\mathscr{C}$.*

Theorem (Mar., Neven, Schwentick 2004)

There is a class of regular expressions $\mathscr{X}$ such that

- **INTERSECTION** *for $\mathscr{X}$ is* **NP***-complete;*
- **INTERSECTION** *for single-type EDTD($\mathscr{X}$) is* **EXPTIME***-complete.*

Remark: **INTERSECTION** for deterministic REs is **PSPACE**-complete.

Focus on Regular Expressions

What to remember?

- Decision problems for XML Schema translate to decision problems for regular expressions.

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- Decision problems for XML Schema translate to decision problems for regular expressions.

What regular expression classes are interesting?

Regular expressions that occur in schemas!

- A *base symbol* is a regular expression w, $w?$, or w^* where w is a non-empty string;
- A *factor* is of the form e, $e?$, e^+, or e^* where e is a disjunction of base symbols.
- A *CHAin Regular Expression (CHARE)* is ε, $\emptyset$, or a sequence $f_1 \cdots f_k$ of factors.

[Bex,Neven,Van den Bussche 2004]: $> 90\%$ of expressions in practical DTDs or XSDs are *CHAREs*

Regular Expression Analysis Revisited

Fragment	CONTAINMENT	EQUIVALENCE	INTERSECTION
a, a^+	in **PTIME** (DFA!)	in **PTIME**	in **PTIME**
a, a^*	**coNP**	in **PTIME**	**NP**
$a, a?$	**coNP**	in **PTIME**	**NP**
$a, (+a)^*$	**PSPACE**	in **PSPACE**	**NP**
$all - \{(+w)^*, (+w)^+\}$	**PSPACE**	in **PSPACE**	**NP**
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RE	**PSPACE**	**PSPACE**	**PSPACE**

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RE	**PSPACE**	**PSPACE**	**PSPACE**

Observation

Not many **PTIME** results. . .

What Regular Expressions are Allowed in Schemas?

Counting and shuffle

- Numerical occurrence operator ($\#$): $(a^{[4,5]}(b+c^*)^7)$
- shuffle operator $(a\&b = \{ab, ba\})$

Theorem (Mayer, Stockmeyer 1994)

CONTAINMENT *and* **EQUIVALENCE** *for RE(&) is* **EXPSPACE**-*complete*

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Theorem (Gelade, Mar., Neven 2007)

CONTAINMENT *and* **EQUIVALENCE** *is* **EXPSPACE***-complete for*

- *RE(#) and*
- *RE(#,&)*

On the Search for more **PTIME** fragments

Theorem (Ghelli, Colazzo, Sartiani 2007)

CONTAINMENT *is in* **PTIME** *for conflict-free regular expressions*

Conflict-free

- counting and interleaving allowed!

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Conflict-free

- counting and interleaving allowed!
- single occurrence
- Kleene star only applied to disjunctions single symbols

Outline

Complementing schemas

Schema Complementation

- I have a schema S which I update to S'
- What are the documents I admitted in S, but not in S' anymore?

This should be $L(S) - L(S') = L(S) \cap \overline{L(S')}$

Complementing regular expressions

Given a regular expression r, define a regexp for $\overline{L(r)}$.

Naive approach: transform to an NFA, determinize, complement, and transform again to a regular expression (2EXPTIME)

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Lemma [Gelade and Neven 2008]

For every n, there is a regular expression r of size $\mathcal{O}(n)$, such that any regular expression defining $\overline{L(r)}$ must be of size $\Omega(2^{2^n})$

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Idea

- Ehrenfeucht, Zeiger (1974): There is a class of DFAs K_n whose smallest equivalent regular expression is at least 2^n. (States $= \{1, \ldots, n\}$, edges between i and j labeled with $a_{i,j}$)
- Generalize this theorem to four-letter alphabets
- Construct r of size $\mathcal{O}(n)$ for $\overline{K_{2^n}}$

Outline

Schema Minimization

Schema Minimization

Given a schema D, compute the smallest equivalent schema D'

Why relevant?

- Recall: Query Optimization
- Input: Queries Q_1, Q_2, and a schema D

Smaller schema improves the run-time of the query optimization problems!

Schema Minimization

Minimization is typically studied on automata models

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What's the deterministic automata model for XML?

- single-type EDTDs with DFAs?
- deterministic unranked tree automata?

Schema Minimization

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What's the deterministic automata model for XML?

- single-type EDTDs with DFAs? $\approx$ top-down det.
- deterministic unranked tree automata? $\approx$ bottom-up det.

Single-type EDTD Minimization

Theorem (Mar., Niehren 2005)

- *Single-type EDTD with DFA Minimization is in* **PTIME**
- *Minimal models are unique*

Minimization Algorithm

Reduce the input single-type EDTD
For every pair of states q_1, q_2, decide equivalence
If equivalent, merge q_1 and q_2
In the resulting EDTD, minimize each DFA

Unranked Tree Automaton Minimization

(Brüggemann-Klein, Murata, Wood 2001)

A bottom-up unranked tree automaton is *deterministic* if for every pair of rules $a(L_1) \to q_1$ and $a(L_2) \to q_2$,

$$L_1 \cap L_2 = \emptyset$$

Additional requirement: L_1, L_2 represented by DFAs

Theorem (Mar., Niehren 2005)

MINIMIZATION *is* **NP**-*complete for deterministic unranked tree automata*

Unranked Tree Automaton Minimization

For the right definition of bottom-up determinism:

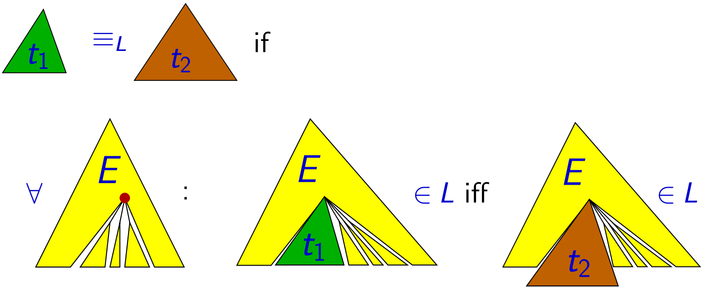
Theorem (Mar., Niehren 2005)

- **MINIMIZATION** *is in* **P** *for bottom-up deterministic tree automata*
- *the Myhill-Nerode theorem for unranked tree languages holds*

Myhill-Nerode for Unranked Tree Automata

For tree language L, define relation $\equiv_L$ on trees

Definition



$\equiv_L$ is an equivalence relation on unranked trees

Myhill-Nerode for Unranked Tree Automata

Theorem (Myhill-Nerode for Unranked Trees (Mar., Niehren 2005))

Let L be an unranked tree language.
The following are equivalent:

- *L is regular*
- *$\equiv_L$ has finitely many equivalence classes*

Moreover, the equivalence classes of $\equiv_L$ correspond to states of minimal (new) bottom-up deterministic unranked TA for L

Question

How much non-determinism can be admitted for **PTIME** minimization?

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DFA $\rightarrow$ unambiguous FA **MINIMIZATION** *is* **NP***-complete*

Theorem (Malcher 2003)

MINIMIZATION *is* **NP***-complete for*

- *NFAs with fixed branching (≥ 3)*
- *NFAs with at least two start states*

Back to the Basics

NFA Minimization

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Question Revisited

Can there be any non-determinism at all?

Definition (δNFA)

The class of NFAs that

- have at most one pair (q, a) such that $(q, a) \to q_1$ and $(q, a) \to q_2$
- are unambiguous
- do not loop

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The class of NFAs that

- have at most one pair (q, a) such that $(q, a) \to q_1$ and $(q, a) \to q_2$
- are unambiguous
- do not loop

Theorem (Björklund, Mar., ICALP 2008)

For every class $\mathscr{C}$ of NFAs such that $\delta NFA \subseteq \mathscr{C}$:

$DFA \to \mathscr{C}$ **MINIMIZATION** *is* **NP**-*hard*

Conclusion and Outlook

XML and Formal Languages are great for cross-fertilization

- Many problems in XML research are solved through FLT techniques
- XML research poses interesting questions for FLT

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- Many problems in XML research are solved through FLT techniques
- XML research poses interesting questions for FLT

So, ...

- if you like formal language theory, but also want a PODS/ICDT paper

  have a look at XML

- if you like formal language theory, and you want more formal language theory

  have a look at XML