The Typechecking Problem for XML Transformations: Methods and Formal Models

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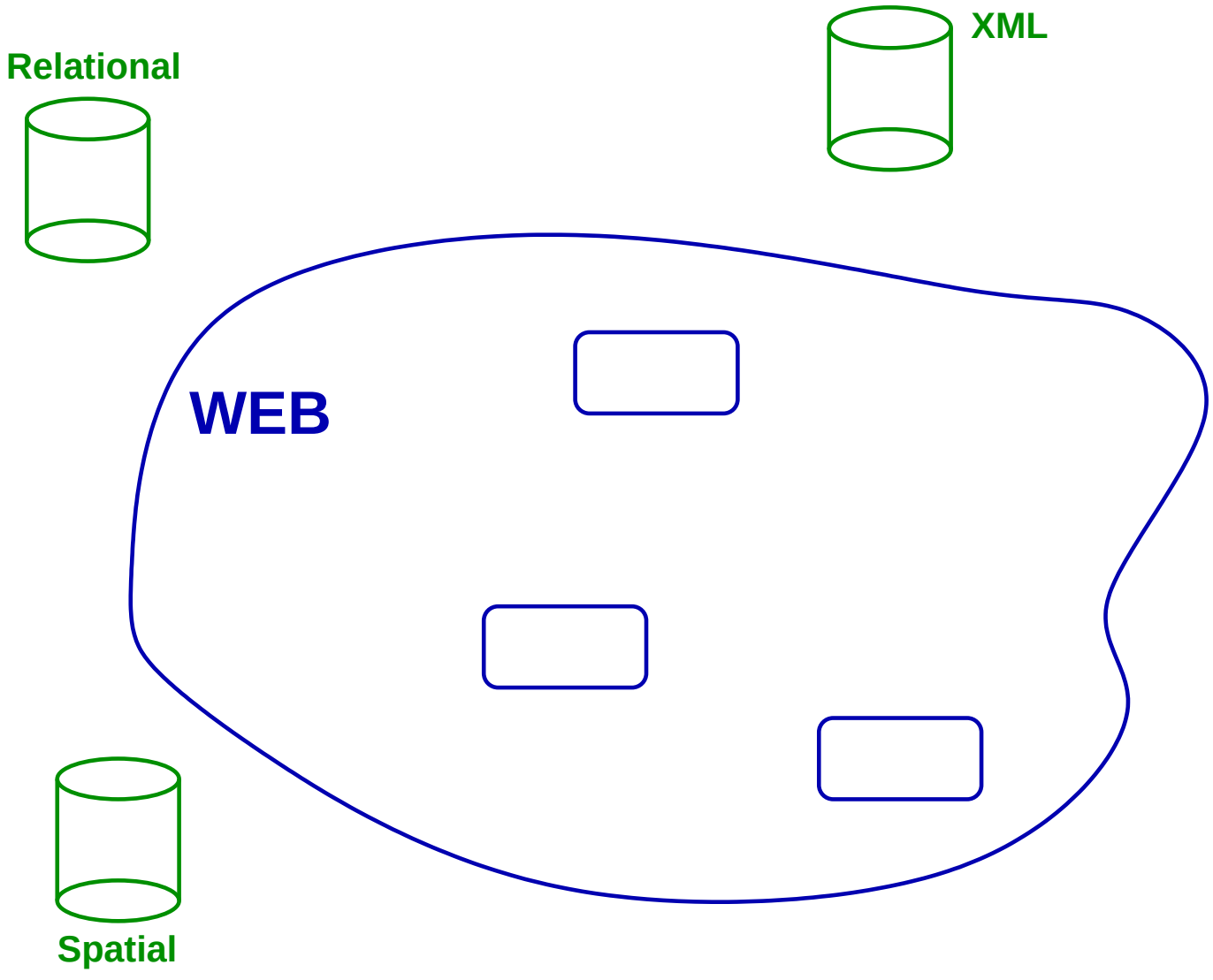
Outline

- What is typechecking?
- Which varieties have been investigated?
- What do I want to discuss?
- Formal models for the typechecking problem
  - XML schema languages
  - XML transformations
- Methods for the typechecking problem
  - Proving upper bounds
  - Proving lower bounds

Data Integration on the Web



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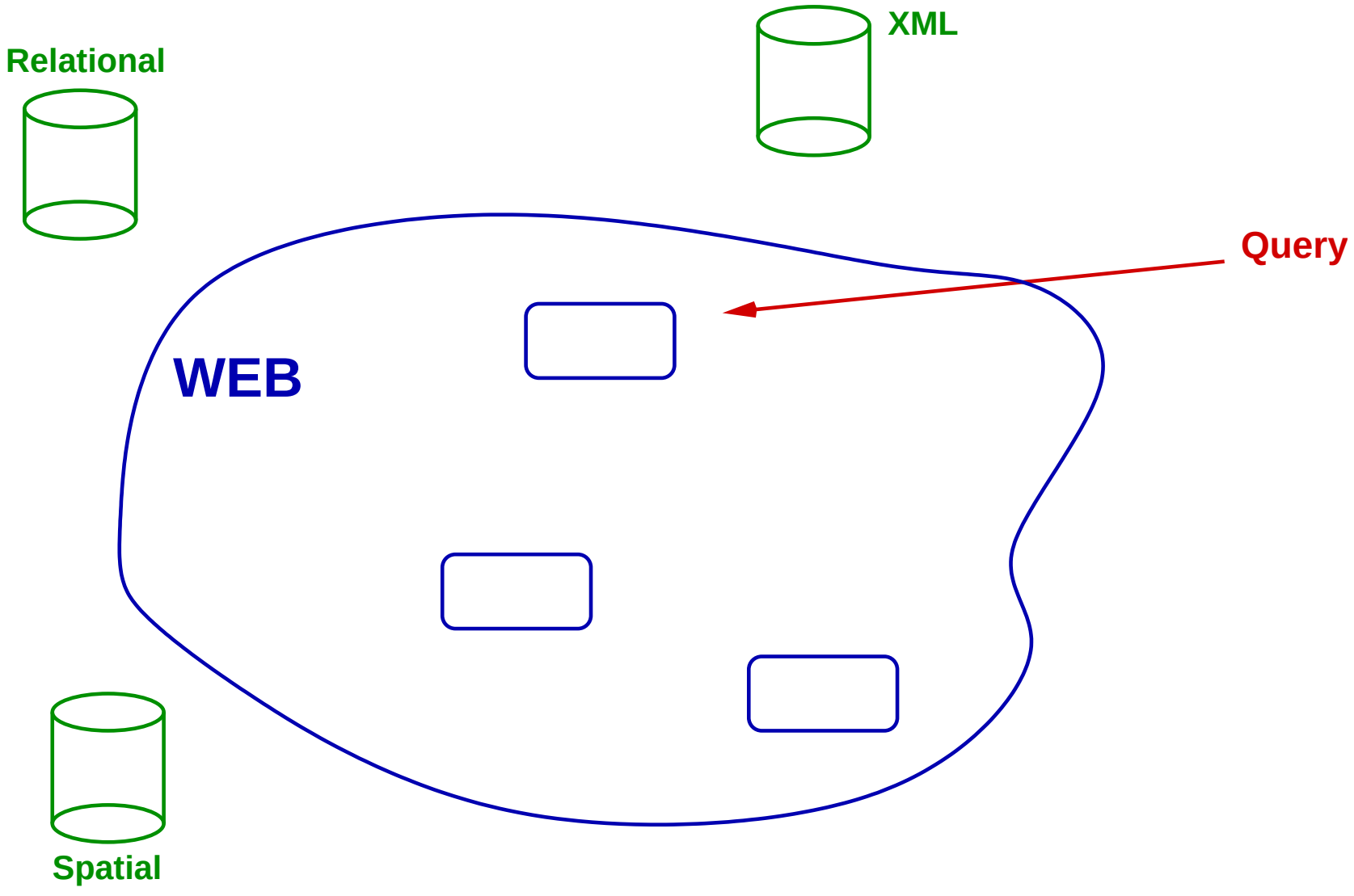

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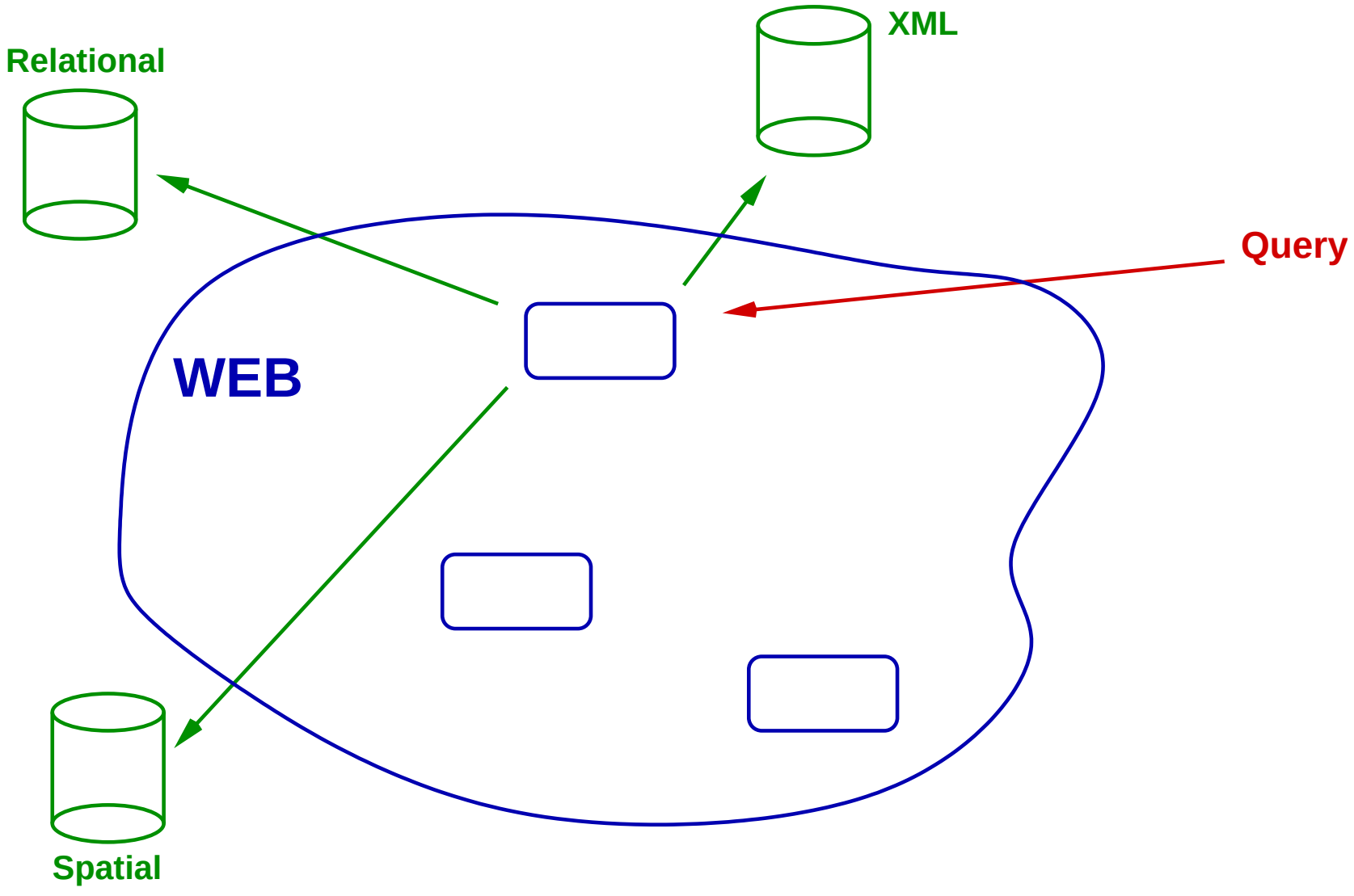

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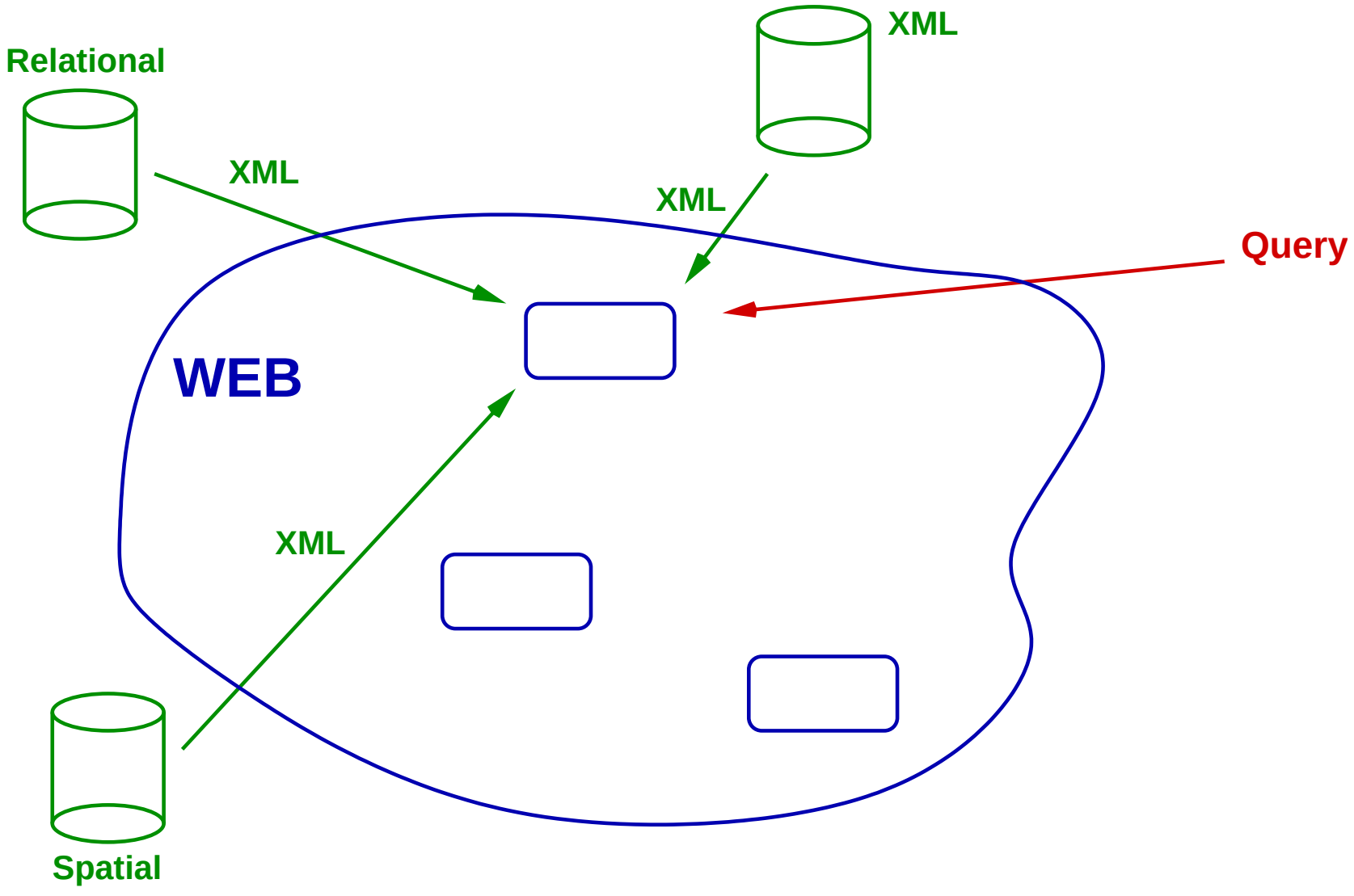

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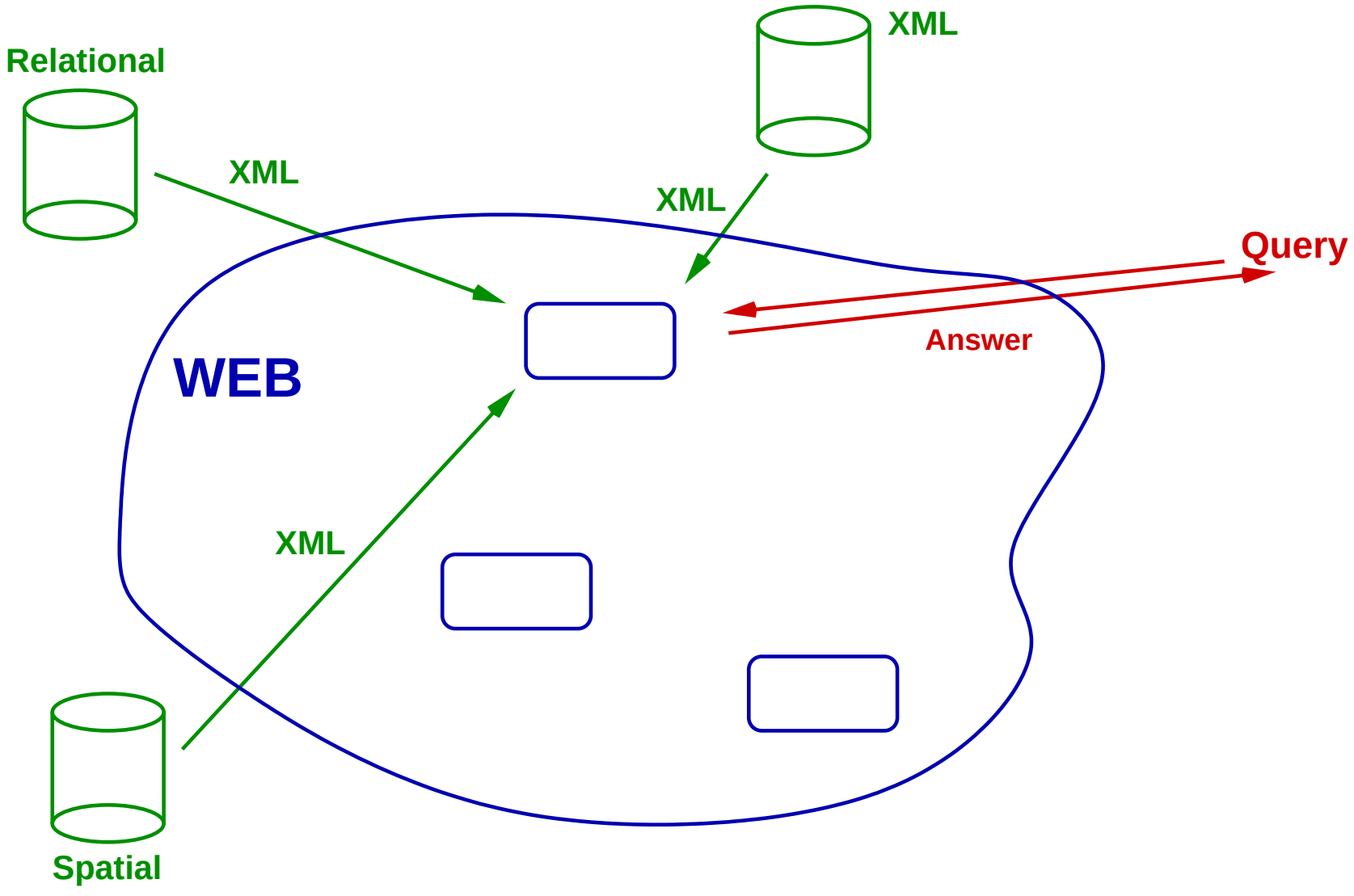

What is Typechecking?

Typechecking:

$$\text{is } \forall t \in S_{\text{in}} : T(t) \in S_{\text{out}}?$$

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Which varieties have been investigated?

Complete vs. Incomplete:

Today, typecheckers are sound but incomplete.
Excellent tutorial on incomplete typecheckers:
[Møller, Schwartzbach, ICDT 05]

Data Values vs. No Data Values:

Complete typechecking quickly turns undecidable when data values are incorporated [Alon et al. 2001]

We focus on complete typechecking where data values are not taken into account

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What do I want to discuss?

Formal Models

. . . for modelling

- XML schema languages and
- XML transformations

Methods

. . . for obtaining upper and lower bounds on the complexity of the typechecking problem

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XML Documents are Trees

```
<store>
    <dvd type="movie">
        <title> "Lola Rennt" </title>
        <price> 11 </price>
        <discount> 6 </discount>
    </dvd>
    <dvd type="movie">
        <title> "Goodbye, Lenin!" </title>
        <price> 20 </price>
    </dvd>
</store>
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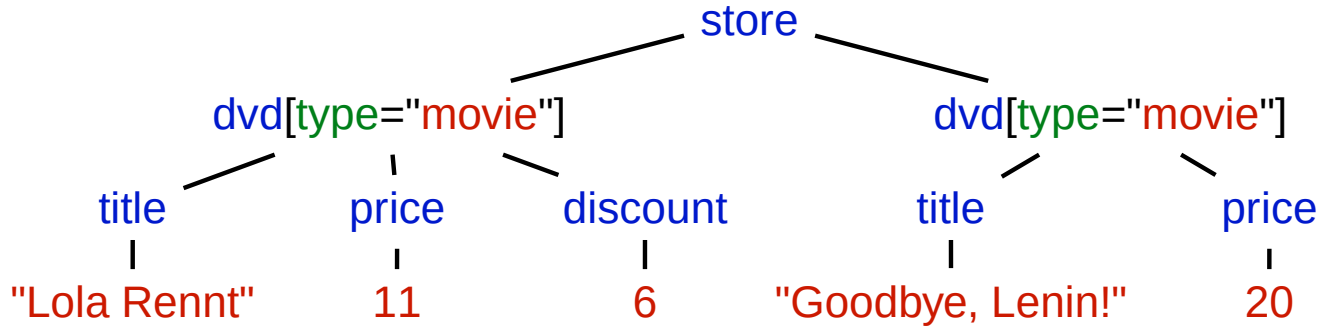

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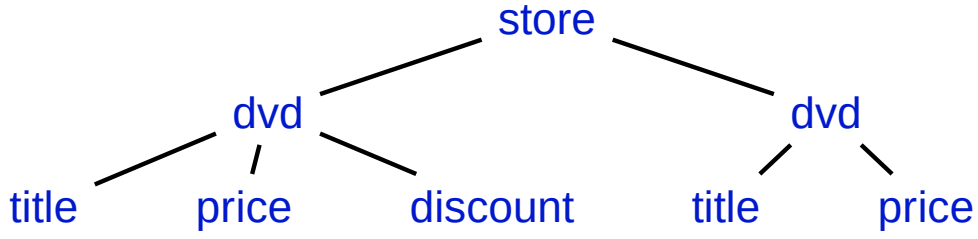

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XML Schema Languages: DTDs

- DTDs (Document Type Definitions):

$$\begin{aligned} \text{store} &\longrightarrow \text{dvd dvd}^* \\ \text{dvd} &\longrightarrow \text{title price discount?} \end{aligned}$$

XML Schema Languages: DTDs

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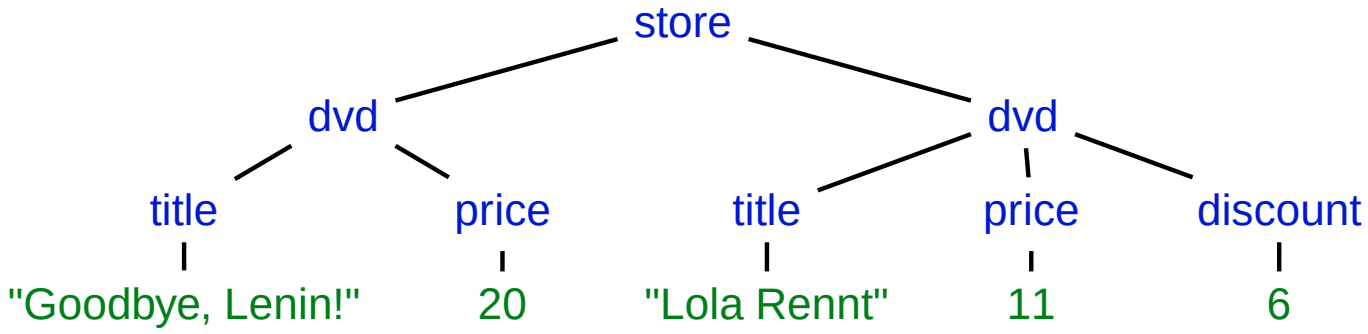

Regular Tree Languages

Extended DTDs (EDTDs) [Papak., Vianu 2000]:
$\equiv$ tree automata on unranked trees

$$\begin{aligned} \text{store} &\rightarrow (\text{dvd}^1)^* \ \text{dvd}^2 \ (\text{dvd}^2)^* \\ \text{dvd}^1 &\rightarrow \text{title price} \\ \text{dvd}^2 &\rightarrow \text{title price discount} \end{aligned}$$

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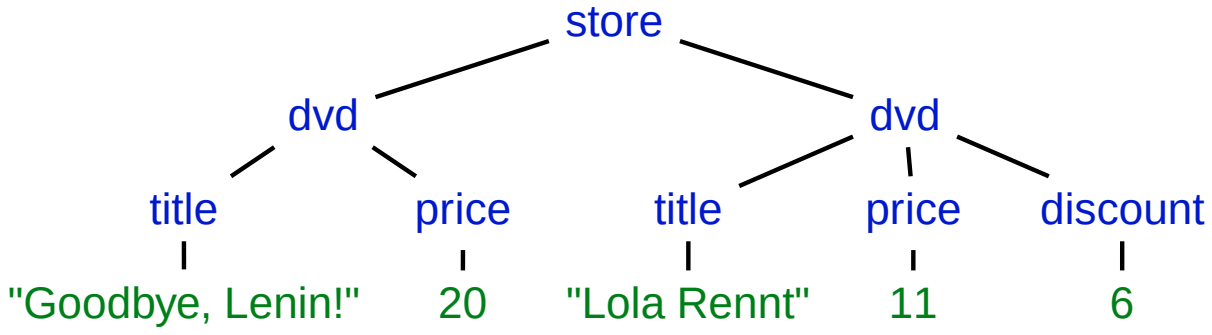

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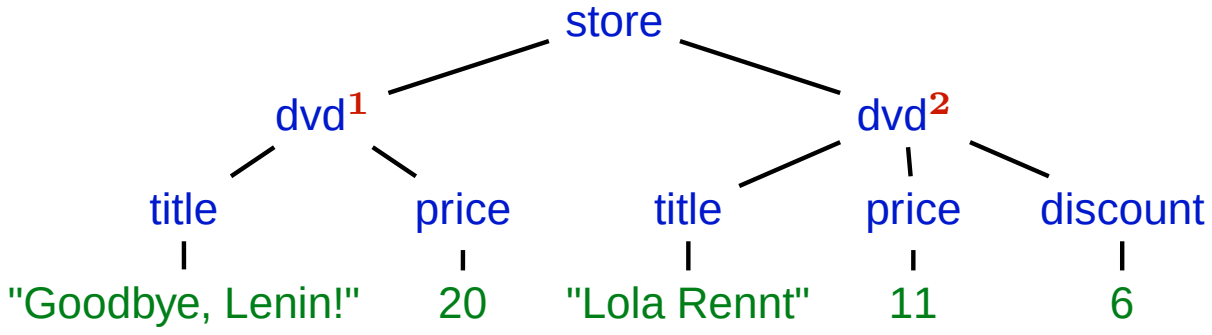

XML Schema

Single-type EDTDs (stEDTDs) [Murata et al., 2001]:

. . . capture the expressive power of XML Schema

Different types for a label in the same rhs are not allowed!

Example:

$\text{store} \longrightarrow (\text{dvd}^{\mathbf{1}})^* \ \text{dvd}^{\mathbf{2}} \ (\text{dvd}^2)^*$	not allowed
$\text{dvd}^1 \longrightarrow \text{title}^2 \ \text{price}^3$	allowed

XML Schema

Single-type EDTDs (stEDTDs) [Murata et al., 2001]:

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\begin{array}{lcl}
\text{store} & \longrightarrow & \text{regulars}^* \ \text{discounts} \ \text{discounts}^* \\
\text{regulars} & \longrightarrow & \text{dvd}^1 \\
\text{discounts} & \longrightarrow & \text{dvd}^2 \\
\text{dvd}^1 & \longrightarrow & \text{title price} \\
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\end{array}
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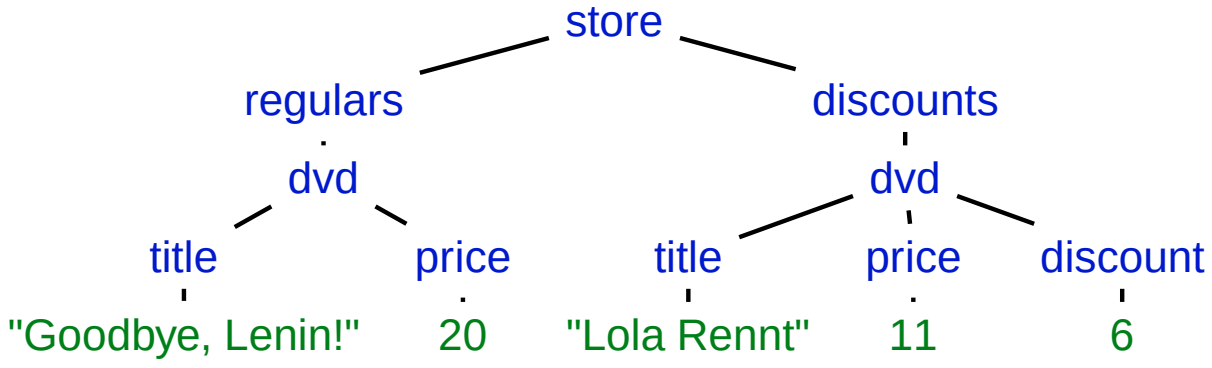

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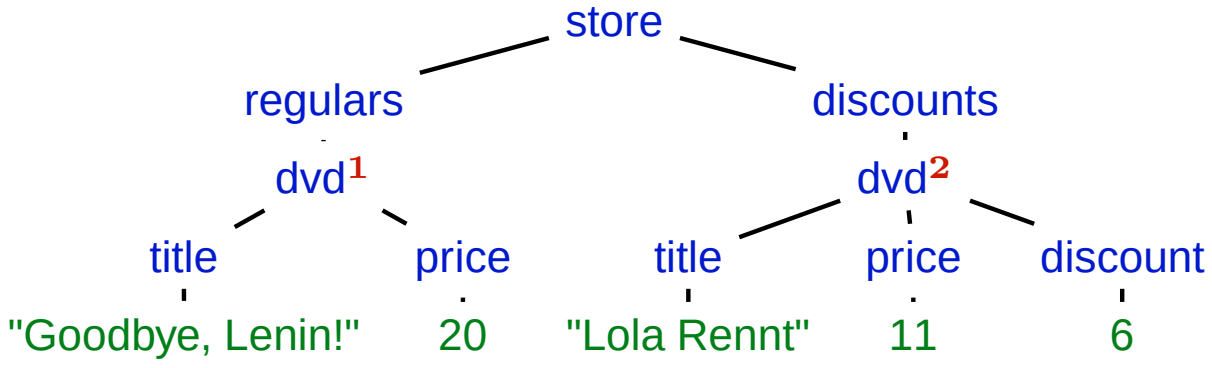

Note: DTD $\subsetneq$ single-type EDTD $\subsetneq$ EDTD

stEDTDs: Alternative Characterizations

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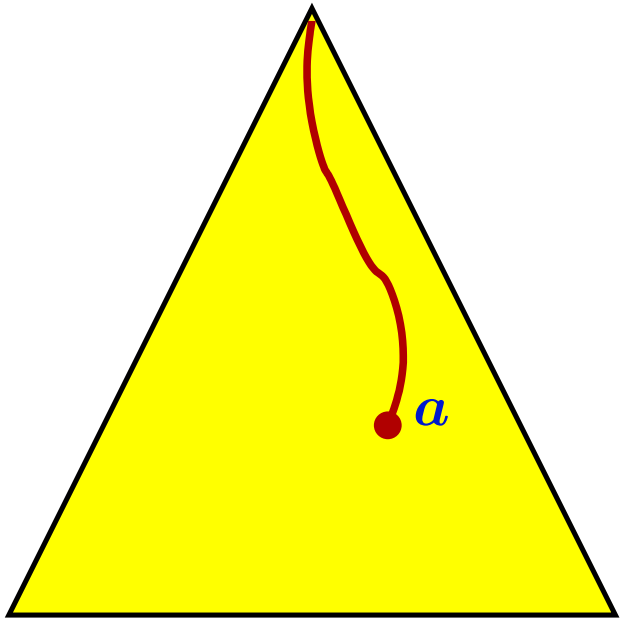
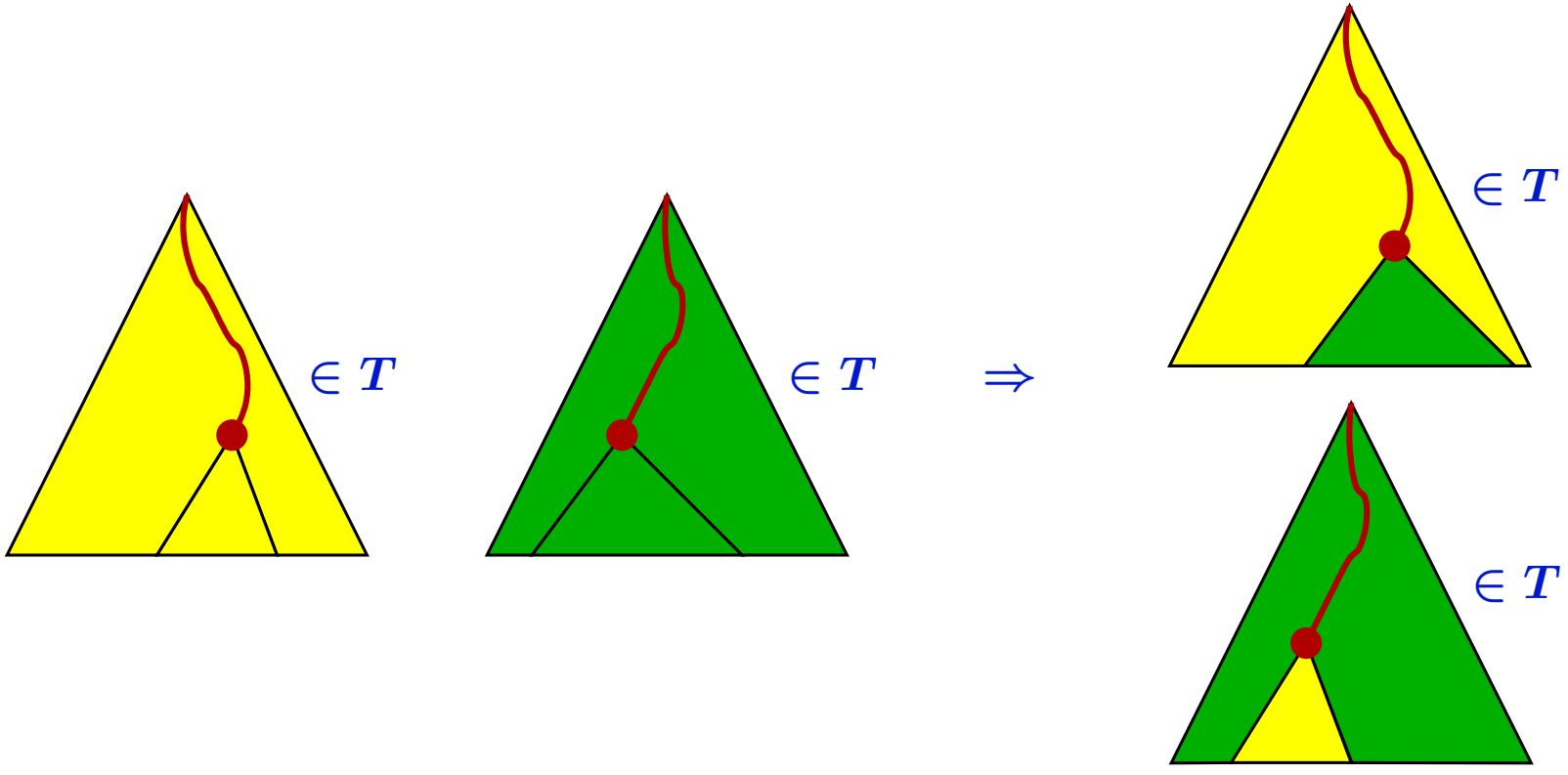
The Ancestor-string:

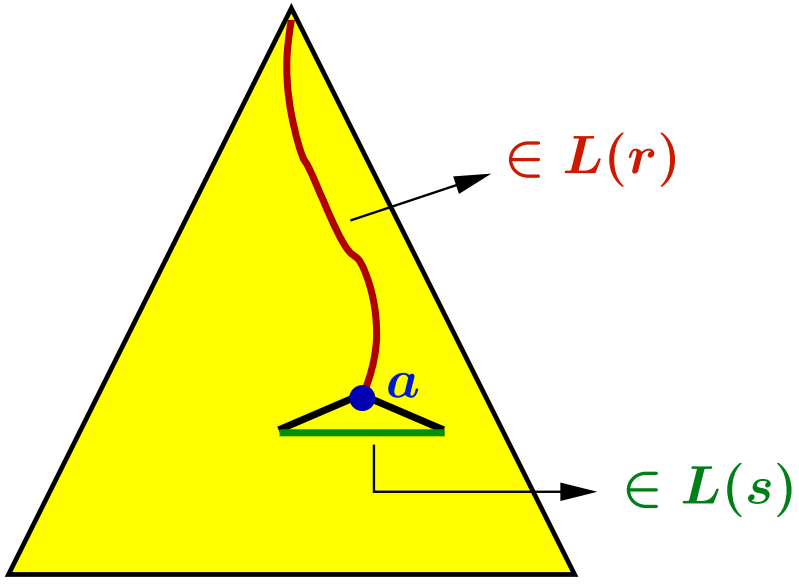


Ancestor-Guarded Subtree Exchange

T a regular tree language



Ancestor-Guarded DTDs



Ancestor-guarded DTD consists of triples $(r, a) \longrightarrow s$

The Equivalence

Let T be a regular tree language

The following are equivalent:

- T is definable by a single-type EDTD
- T is closed under ancestor-guarded subtree exchange
- T is definable by an ancestor-guarded DTD

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DTDs can be characterized analogously

Closure Properties

Are these schema languages closed under union, difference, complement or intersection?

	union	difference	complement	intersection
DTD	no	no	no	yes
Single-type	no	no	no	yes
EDTD	yes	yes	yes	yes

Non-closure proofs are easy by the presented subtree-exchange properties

Closure proofs are by direct construction

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Simple Tree Transducers

[M., Neven 2003]

"A finite automaton running on all paths from root to leaf", while producing output

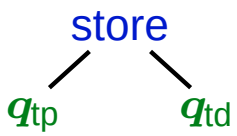
At each point in the computation it either

- stops; or
- continues in all the children of the current node

It can continue computation in multiple states (i.e. copying).

GOAL: Model simple restructuring transformations

Simple Tree Transducers: Example

$(q_o, \text{store}) \rightarrow$ 

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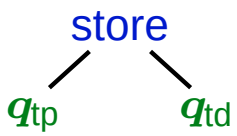
$(q_0, \text{store}) \rightarrow$ 

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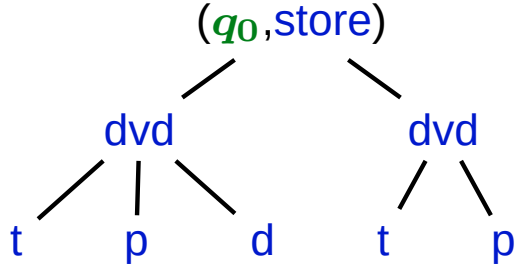

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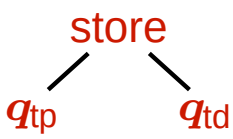
$(q_0, \text{store}) \rightarrow$ 

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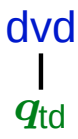
$(q_{td}, \text{dvd}) \rightarrow$ 

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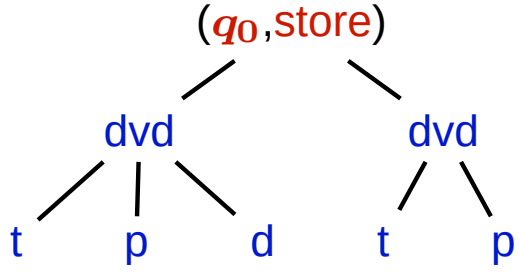

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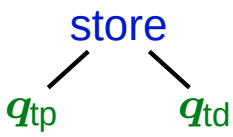
$(q_0, \text{store}) \rightarrow$ 

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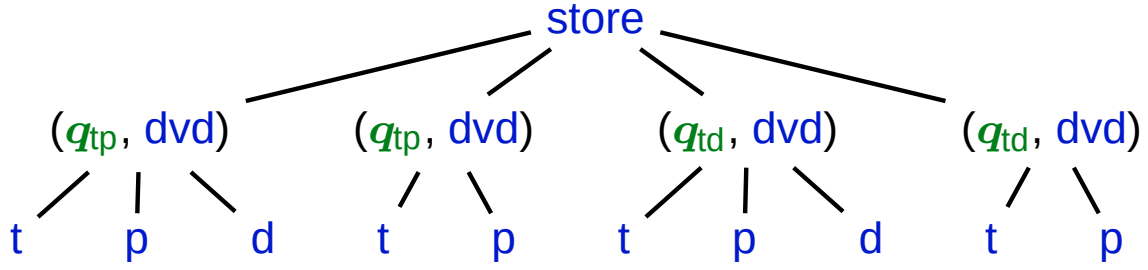

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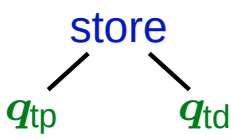
$(q_0, \text{store}) \rightarrow$ 

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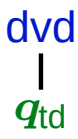
$(q_{td}, \text{dvd}) \rightarrow$ 

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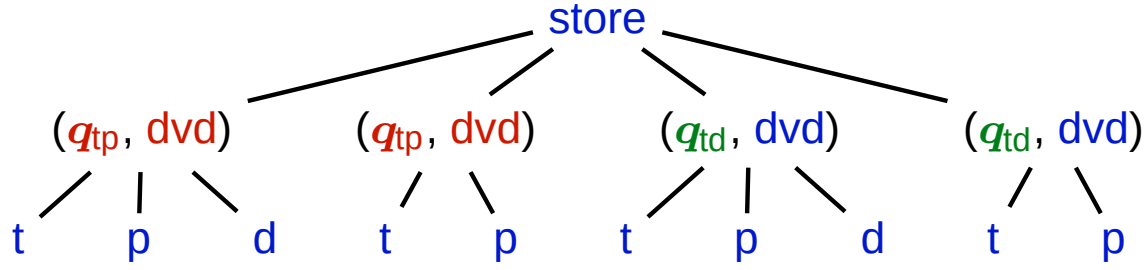

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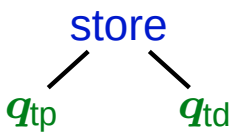
$(q_0, \text{store}) \rightarrow$ 

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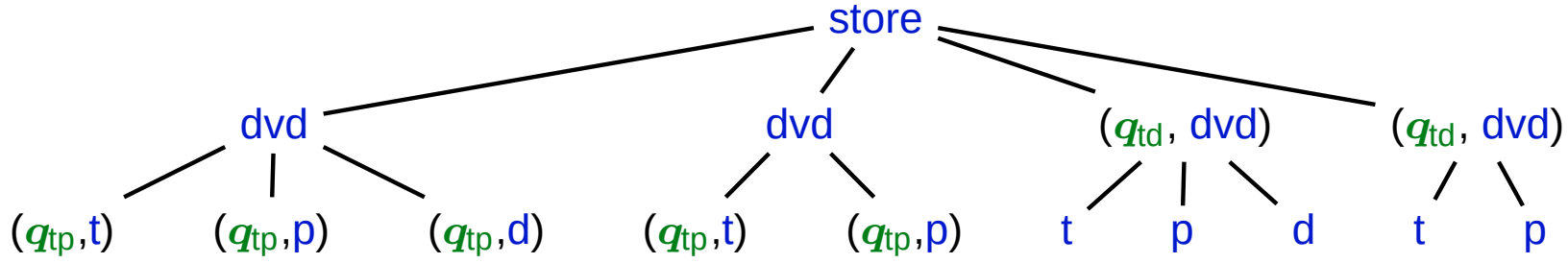

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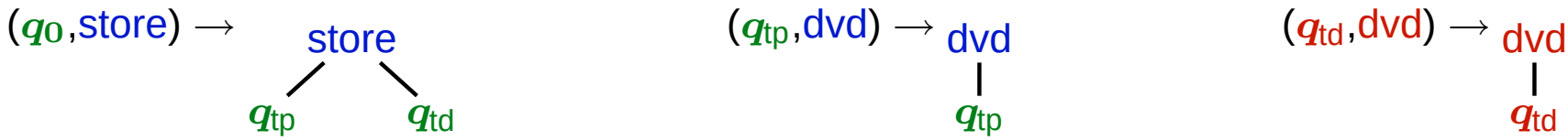

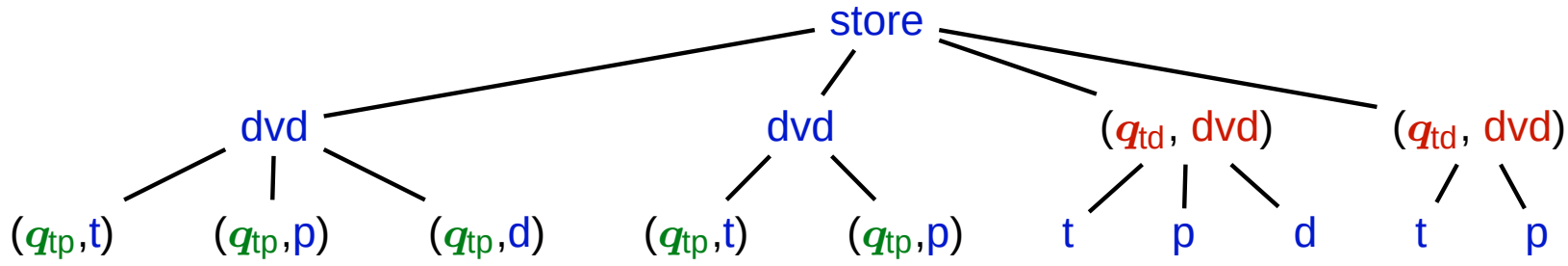

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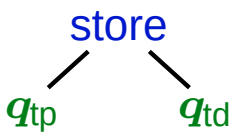
$(q_0, \text{store}) \rightarrow$ 

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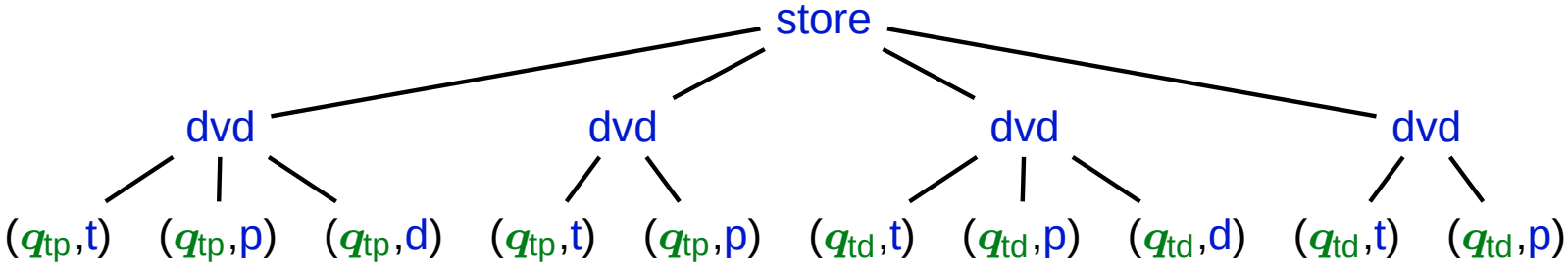

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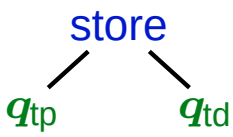
$(q_0, \text{store}) \rightarrow$ 

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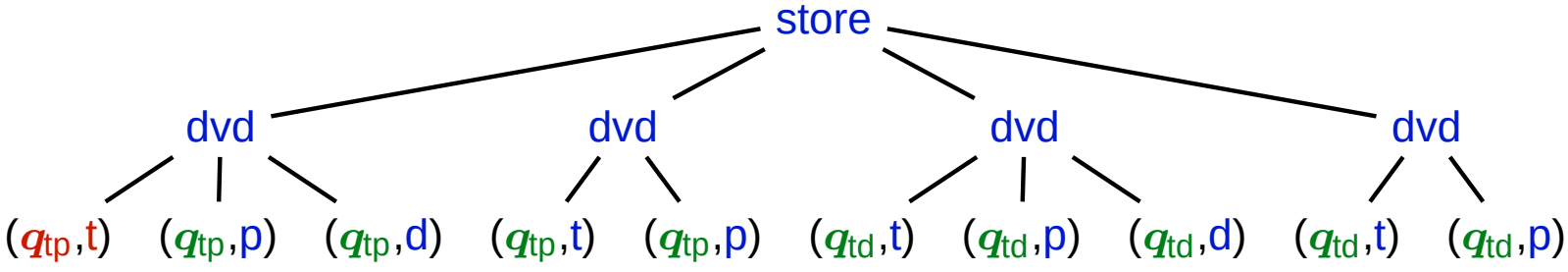

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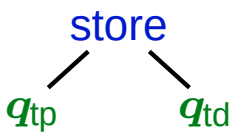
$(q_0, \text{store}) \rightarrow$ 

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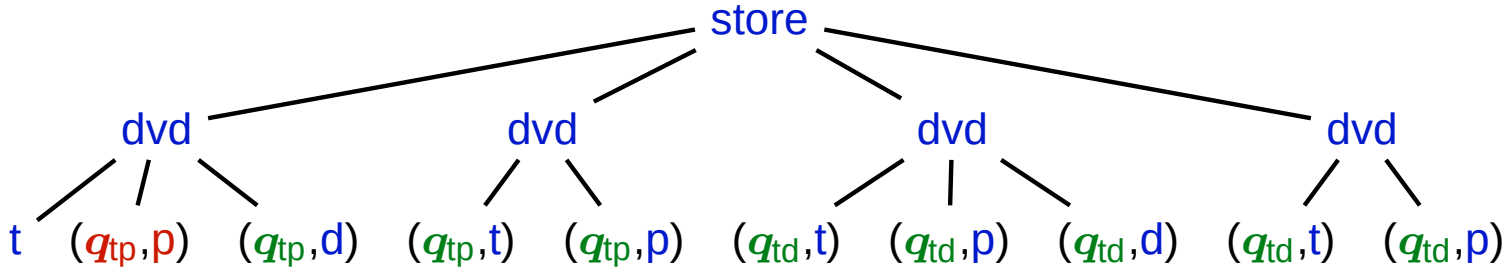
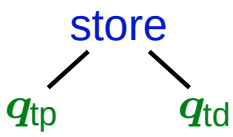


Simple Tree Transducers: Example

$(q_0, \text{store}) \rightarrow$ 

$(q_{tp}, \text{dvd}) \rightarrow$ 

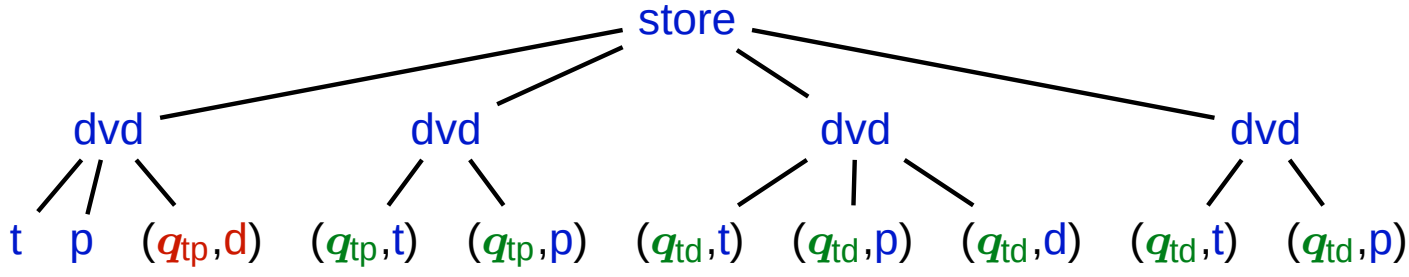
$(q_{td}, \text{dvd}) \rightarrow$ 

$(q_{tp}, \text{t}) \rightarrow \text{t}$

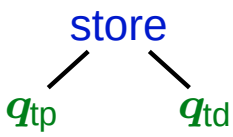
$(q_{tp}, \text{p}) \rightarrow \text{p}$

$(q_{td}, \text{t}) \rightarrow \text{t}$

$(q_{td}, \text{d}) \rightarrow \text{d}$



Simple Tree Transducers: Example

$(q_0, \text{store}) \rightarrow$ 

$(q_{tp}, \text{dvd}) \rightarrow$ 

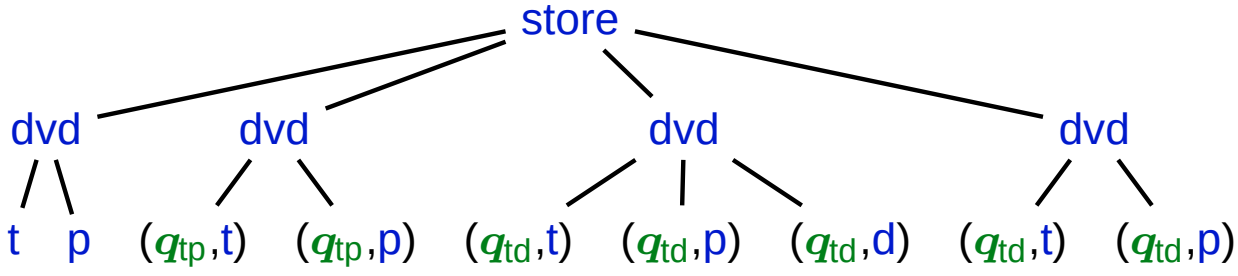
$(q_{td}, \text{dvd}) \rightarrow$ 

$(q_{tp}, \text{t}) \rightarrow \text{t}$

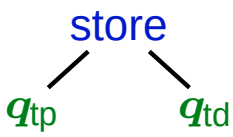
$(q_{tp}, \text{p}) \rightarrow \text{p}$

$(q_{td}, \text{t}) \rightarrow \text{t}$

$(q_{td}, \text{d}) \rightarrow \text{d}$



Simple Tree Transducers: Example

$(q_0, \text{store}) \rightarrow$ 

$(q_{tp}, \text{dvd}) \rightarrow$ 

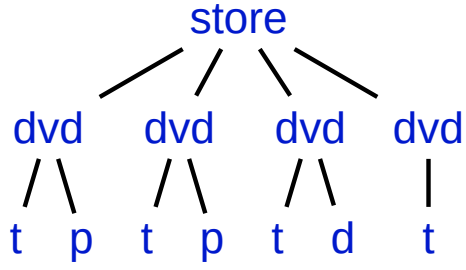
$(q_{td}, \text{dvd}) \rightarrow$ 

$(q_{tp}, \text{t}) \rightarrow \text{t}$

$(q_{tp}, \text{p}) \rightarrow \text{p}$

$(q_{td}, \text{t}) \rightarrow \text{t}$

$(q_{td}, \text{d}) \rightarrow \text{d}$



Macro Tree Transducers

Macro tree transducers [Engelfriet, Vogler 1985]

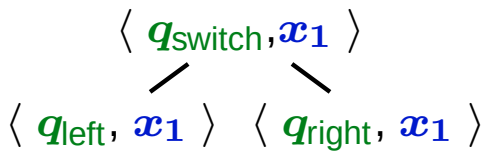
. . . are much more powerful than Simple Tree Transducers

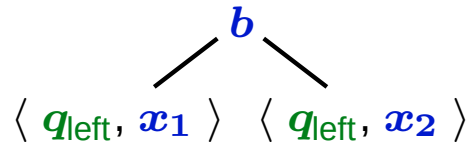
. . . normally work on ranked instead of unranked trees

. . . can be seen as a term rewriting system with input variables and output variables

. . . are also a useful tool to obtain complexity upper bounds

Macro Tree Transducers: Example

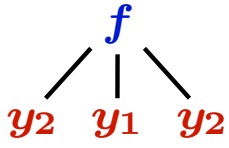
$\langle\, q_{\text{copy}}, a(x_1) \,\rangle \to$ 

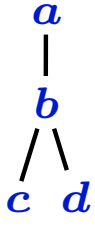
$\langle\, q_{\text{left}}, b(x_1, x_2) \,\rangle \to$ 

$\langle\, q_{\text{left}}, c \,\rangle \to c$

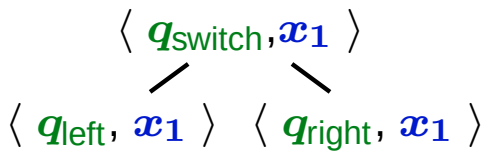
$\langle\, q_{\text{left}}, d \,\rangle \to d$

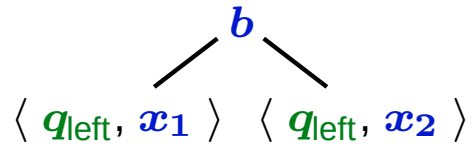
$\langle\, q_{\text{right}}, b(x_1, x_2) \,\rangle \to e$

$\langle\, q_{\text{switch}}, b(x_1, x_2) \,\rangle\, (y_1, y_2) \to$ 



Macro Tree Transducers: Example

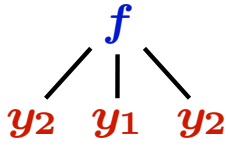
$\langle q_{\text{copy}}, a(x_1) \rangle \to$ 

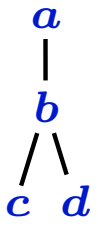
$\langle q_{\text{left}}, b(x_1, x_2) \rangle \to$ 

$\langle q_{\text{left}}, c \rangle \to c$

$\langle q_{\text{left}}, d \rangle \to d$

$\langle q_{\text{right}}, b(x_1, x_2) \rangle \to e$

$\langle q_{\text{switch}}, b(x_1, x_2) \rangle (y_1, y_2) \to$ 

$\langle q_{\text{copy}},$  $\rangle$

Macro Tree Transducers: Example

$\langle\, q_{\text{copy}}, a(x_1) \,\rangle \to$ $\langle\, q_{\text{switch}}, x_1 \,\rangle$ with children $\langle\, q_{\text{left}}, x_1 \,\rangle$ and $\langle\, q_{\text{right}}, x_1 \,\rangle$

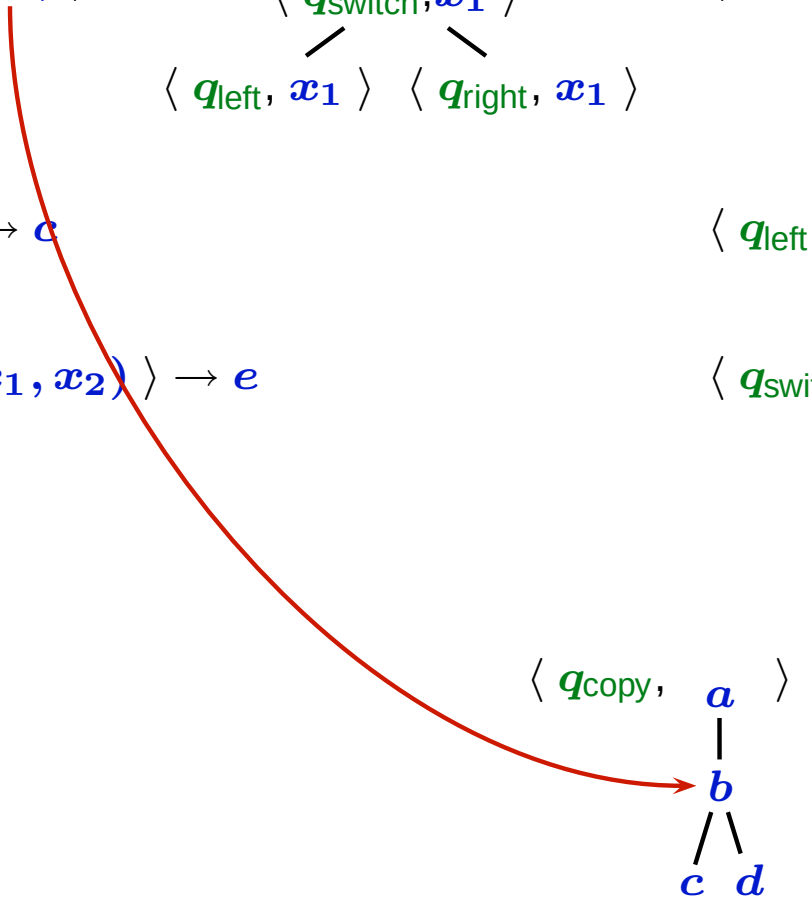
$\langle\, q_{\text{left}}, b(x_1, x_2) \,\rangle \to$ b with children $\langle\, q_{\text{left}}, x_1 \,\rangle$ and $\langle\, q_{\text{left}}, x_2 \,\rangle$

$\langle\, q_{\text{left}}, c \,\rangle \to c$

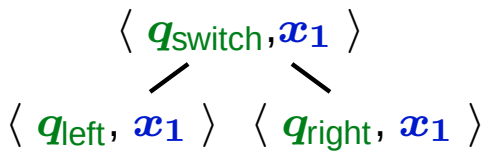
$\langle\, q_{\text{left}}, d \,\rangle \to d$

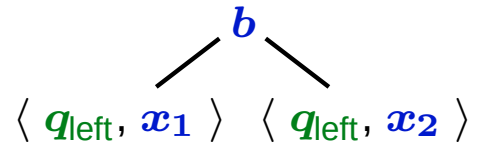
$\langle\, q_{\text{right}}, b(x_1, x_2) \,\rangle \to e$

$\langle\, q_{\text{switch}}, b(x_1, x_2) \,\rangle\, (y_1, y_2) \to$ f with children y_2, y_1, y_2



Macro Tree Transducers: Example

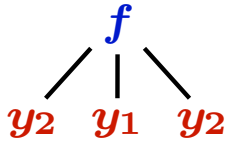
$\langle\, q_{\text{copy}}, a(x_1) \,\rangle \to$ 

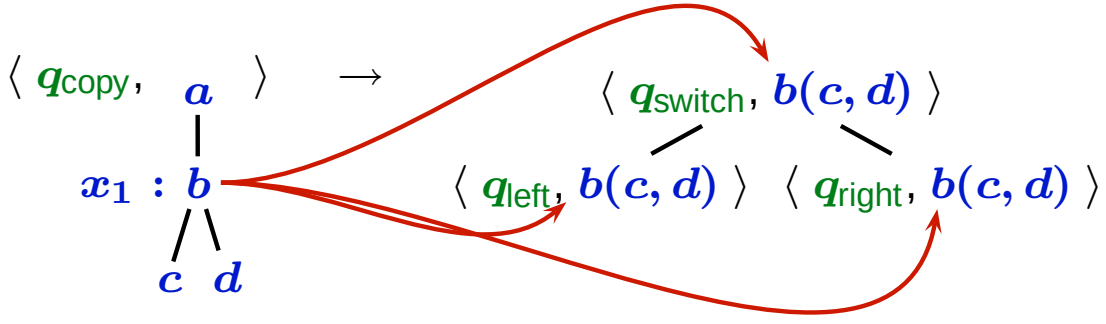
$\langle\, q_{\text{left}}, b(x_1, x_2) \,\rangle \to$ 

$\langle\, q_{\text{left}}, c \,\rangle \to c$

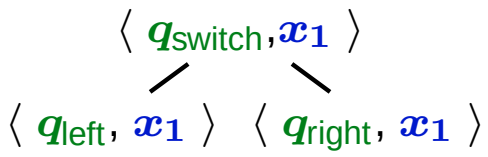
$\langle\, q_{\text{left}}, d \,\rangle \to d$

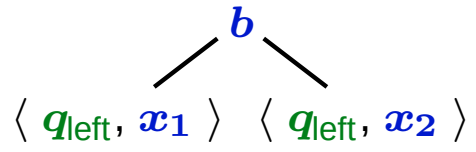
$\langle\, q_{\text{right}}, b(x_1, x_2) \,\rangle \to e$

$\langle\, q_{\text{switch}}, b(x_1, x_2) \,\rangle\, (y_1, y_2) \to$ 



Macro Tree Transducers: Example

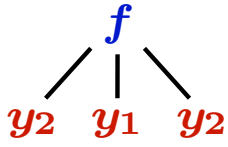
$\langle\, q_{\text{copy}}, a(x_1) \,\rangle \to$ 

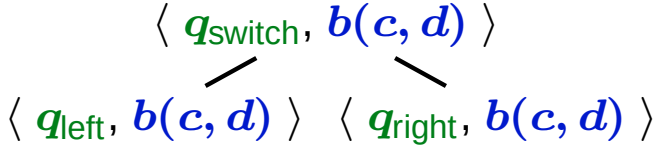
$\langle\, q_{\text{left}}, b(x_1, x_2) \,\rangle \to$ 

$\langle\, q_{\text{left}}, c \,\rangle \to c$

$\langle\, q_{\text{left}}, d \,\rangle \to d$

$\langle\, q_{\text{right}}, b(x_1, x_2) \,\rangle \to e$

$\langle\, q_{\text{switch}}, b(x_1, x_2) \,\rangle\, (y_1, y_2) \to$ 



Macro Tree Transducers: Example

$\langle q_{\text{copy}}, a(x_1) \rangle \to \langle q_{\text{switch}}, x_1 \rangle$ with children $\langle q_{\text{left}}, x_1 \rangle$ and $\langle q_{\text{right}}, x_1 \rangle$

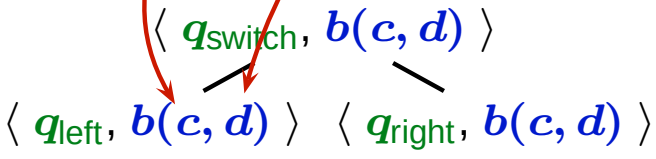
$\langle q_{\text{left}}, b(x_1, x_2) \rangle \to b$ with children $\langle q_{\text{left}}, x_1 \rangle$ and $\langle q_{\text{left}}, x_2 \rangle$

$\langle q_{\text{left}}, c \rangle \to c$

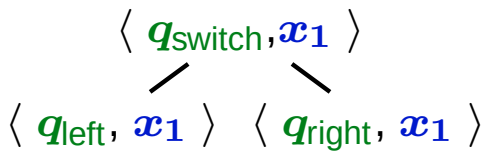
$\langle q_{\text{left}}, d \rangle \to d$

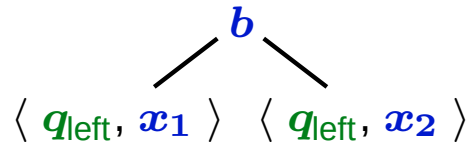
$\langle q_{\text{right}}, b(x_1, x_2) \rangle \to e$

$\langle q_{\text{switch}}, b(x_1, x_2) \rangle (y_1, y_2) \to f$ with children y_2, y_1, y_2



Macro Tree Transducers: Example

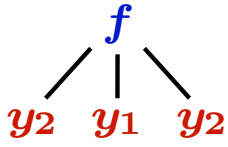
$\langle\, q_{\text{copy}}, a(x_1) \,\rangle \to$ 

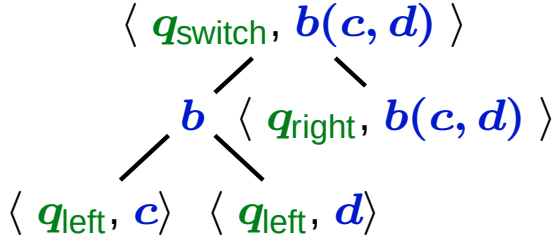
$\langle\, q_{\text{left}}, b(x_1, x_2) \,\rangle \to$ 

$\langle\, q_{\text{left}}, c \,\rangle \to c$

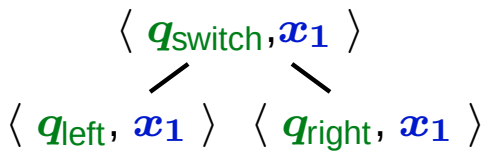
$\langle\, q_{\text{left}}, d \,\rangle \to d$

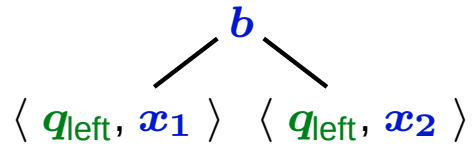
$\langle\, q_{\text{right}}, b(x_1, x_2) \,\rangle \to e$

$\langle\, q_{\text{switch}}, b(x_1, x_2) \,\rangle\, (y_1, y_2) \to$ 



Macro Tree Transducers: Example

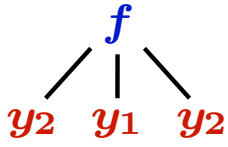
$\langle\, q_{\text{copy}}, a(x_1) \,\rangle \to$ 

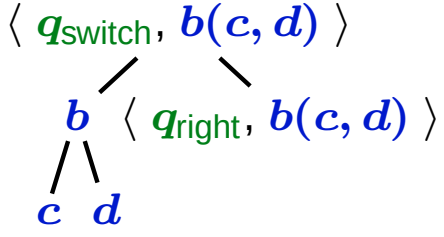
$\langle\, q_{\text{left}}, b(x_1, x_2) \,\rangle \to$ 

$\langle\, q_{\text{left}}, c \,\rangle \to c$

$\langle\, q_{\text{left}}, d \,\rangle \to d$

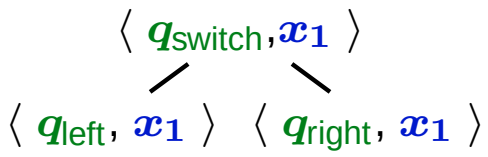
$\langle\, q_{\text{right}}, b(x_1, x_2) \,\rangle \to e$

$\langle\, q_{\text{switch}}, b(x_1, x_2) \,\rangle\, (y_1, y_2) \to$ 

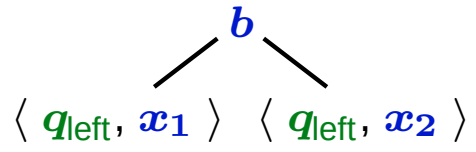


Macro Tree Transducers: Example

$\langle\, q_{\text{copy}}, a(x_1) \,\rangle \to$



$\langle\, q_{\text{left}}, b(x_1, x_2) \,\rangle \to$

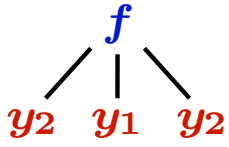


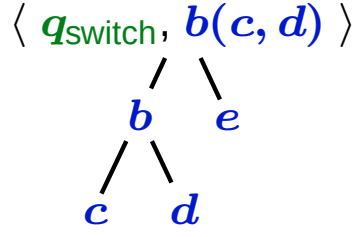
$\langle\, q_{\text{left}}, c \,\rangle \to c$

$\langle\, q_{\text{left}}, d \,\rangle \to d$

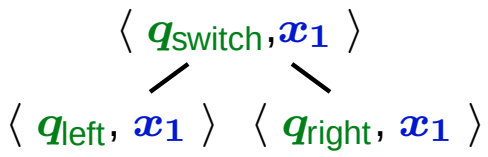
$\langle\, q_{\text{right}}, b(x_1, x_2) \,\rangle \to e$

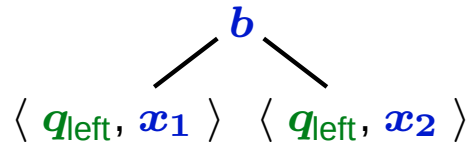
$\langle\, q_{\text{switch}}, b(x_1, x_2) \,\rangle\, (y_1, y_2) \to$





Macro Tree Transducers: Example

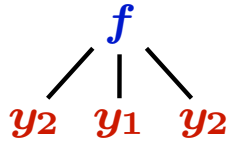
$\langle\, q_{\text{copy}}, a(x_1) \,\rangle \rightarrow$ 

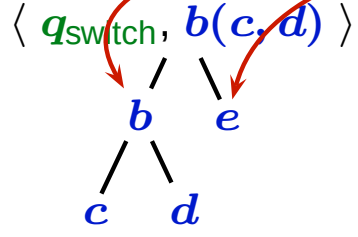
$\langle\, q_{\text{left}}, b(x_1, x_2) \,\rangle \rightarrow$ 

$\langle\, q_{\text{left}}, c \,\rangle \rightarrow c$

$\langle\, q_{\text{left}}, d \,\rangle \rightarrow d$

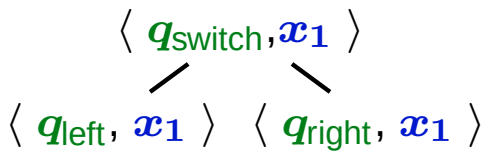
$\langle\, q_{\text{right}}, b(x_1, x_2) \,\rangle \rightarrow e$

$\langle\, q_{\text{switch}}, b(x_1, x_2) \,\rangle\ (y_1, y_2) \rightarrow$ 

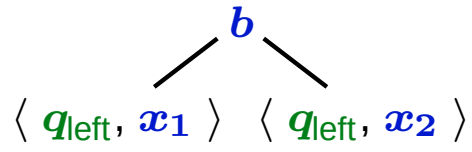


Macro Tree Transducers: Example

$\langle\, q_{\text{copy}}, a(x_1) \,\rangle \rightarrow$



$\langle\, q_{\text{left}}, b(x_1, x_2) \,\rangle \rightarrow$

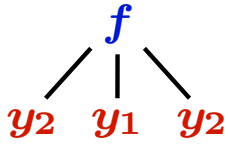


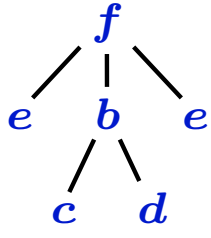
$\langle\, q_{\text{left}}, c \,\rangle \rightarrow c$

$\langle\, q_{\text{left}}, d \,\rangle \rightarrow d$

$\langle\, q_{\text{right}}, b(x_1, x_2) \,\rangle \rightarrow e$

$\langle\, q_{\text{switch}}, b(x_1, x_2) \,\rangle\, (y_1, y_2) \rightarrow$





More Models. . .

. . . are briefly described in the paper:

k-pebble tree transducers: [Milo, Suciu, Vianu 2000]

- Very powerful, designed to model many XML transformation languages
- Typechecking is decidable, non-elementary

TL transformers: [Maneth et al. 2005]

- Strict extension of Simple Tree Transducers
- Typechecking is decidable, space $O(2^{2^{2^{2^{O(n)}}}})$

the query language QL: [Alon et al. 2001]

- Models transformations that compare data values
- Typechecking is undecidable in very restricted cases

Outline

- What is typechecking?
- Which varieties have been investigated?
- What do I want to discuss?
- Formal models for the typechecking problem
  - XML schema languages
  - XML transformations
- Methods for the typechecking problem
  - Proving upper bounds
  - Proving lower bounds

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Emptiness Test

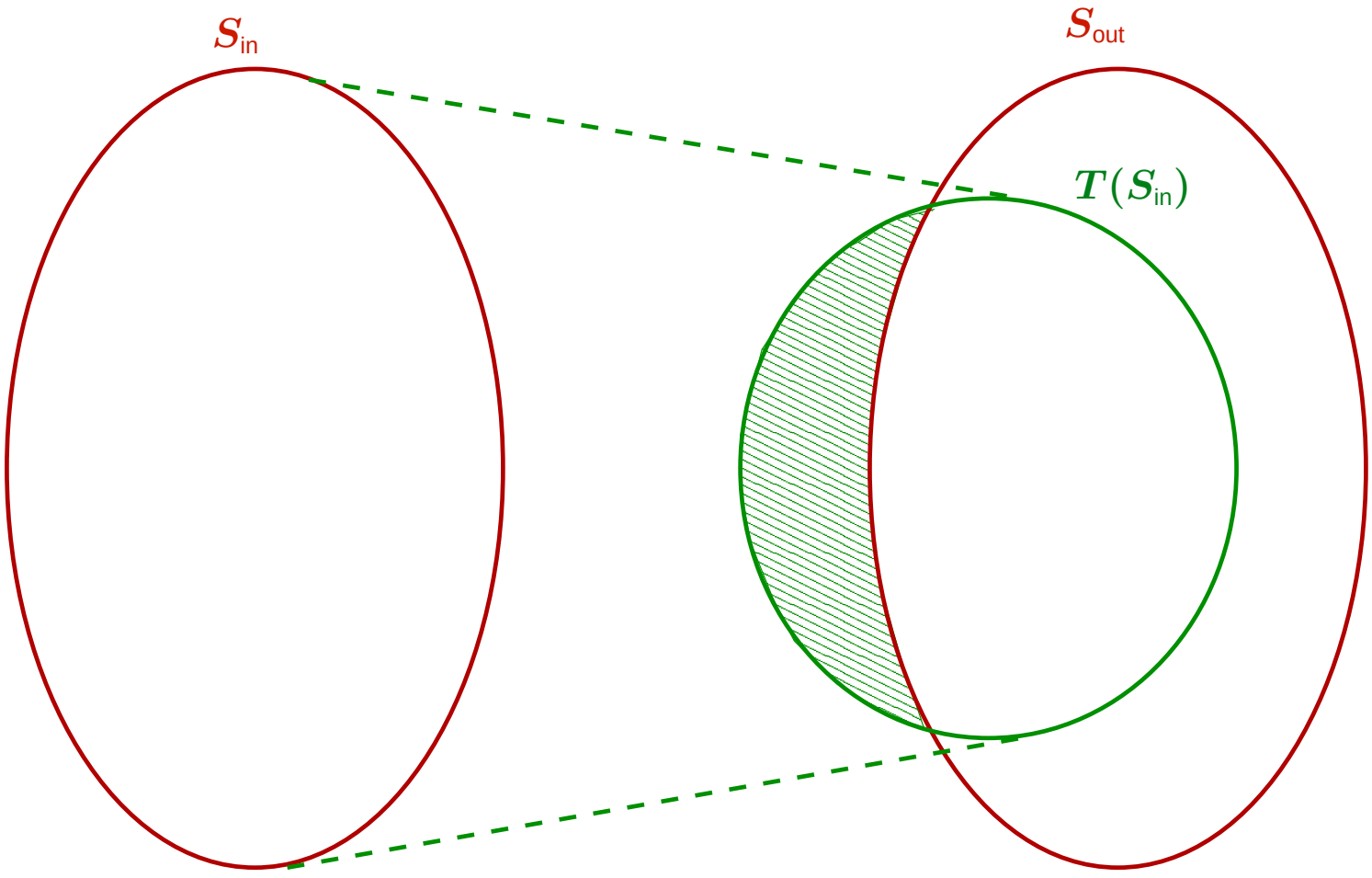
Given a schema S, is $L(S) = \emptyset$?

Reduce entire typechecking problem to
emptiness of one big EDTD (or unranked tree automaton)

Advantage: Emptiness of EDTDs is in **PTIME**

So, the size of the EDTD approximates the time complexity!

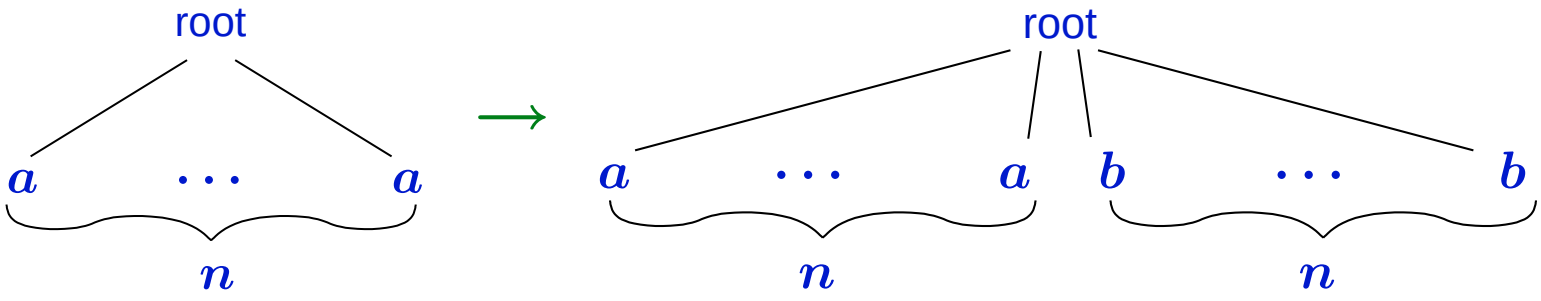
Type Inference



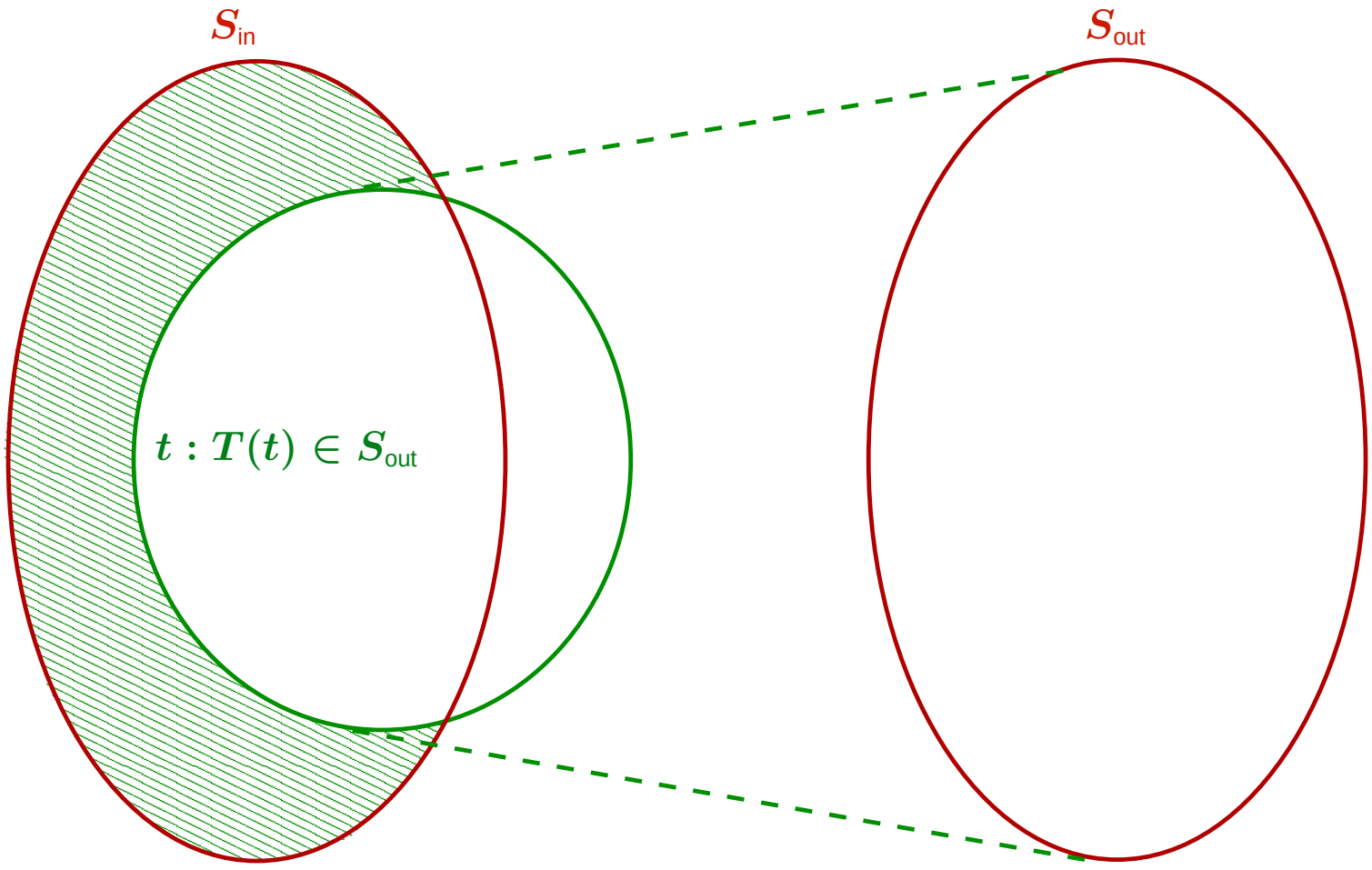
Possible problem: Image is not always regular!

Type Inference:Problem

Image is not always regular: simple example.



Inverse Type Inference



Good news: Pre-image is regular in many cases!

Reduce to Compositions

Write transformation as a composition of MTTs

Inverse type inference through a MTT is doubly exponential

Write transformation as a composition of kPTTs

Inverse type inference through kPTT hyperexponential in k

Seems more aimed towards decidability results

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Inclusion

Is $L(S_{\text{in}}) \subseteq L(S_{\text{out}})$?

Sounds trivial, but is still useful when. . .

. . . input schema is fixed

Universality: $\mathcal{T}_\Sigma \subseteq L(S_{\text{out}})$

. . . output schema is fixed

Emptiness: $L(S_{\text{in}}) \subseteq \emptyset$

Simulation

Simulate automata by the tree transducer

Interesting problem: Intersection Emptiness

Given n automata $A_1, \ldots, A_n$, is

$$L(A_1) \cap \cdots \cap L(A_n) = \emptyset?$$

Simulation

Given n automata $A_1, \ldots, A_n$, is

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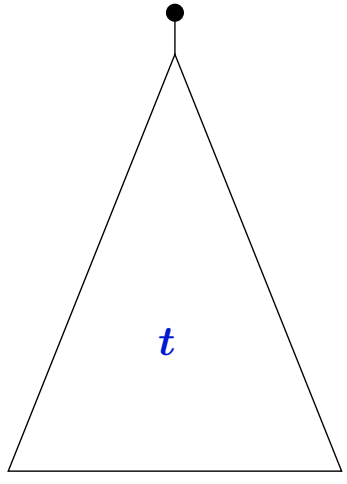

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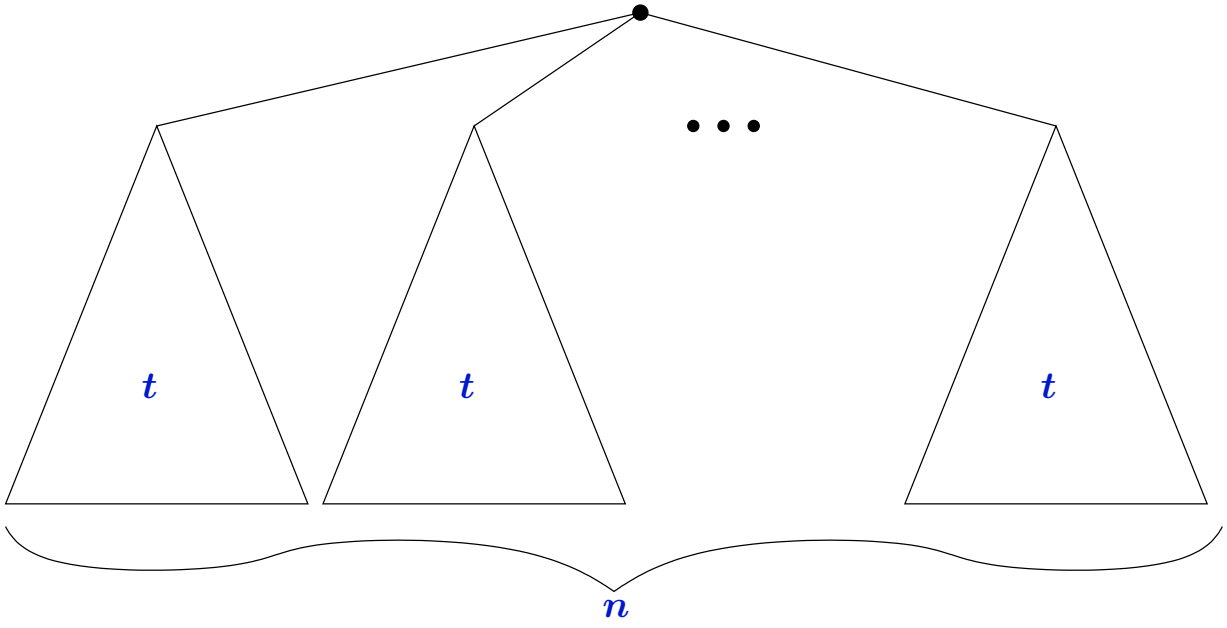

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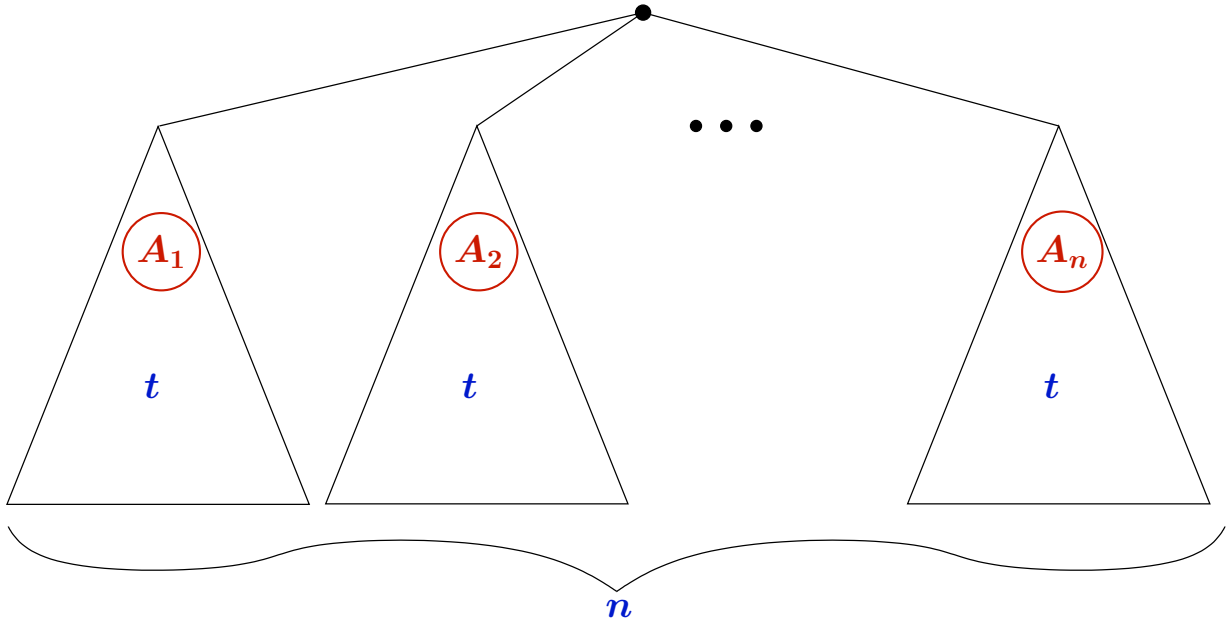

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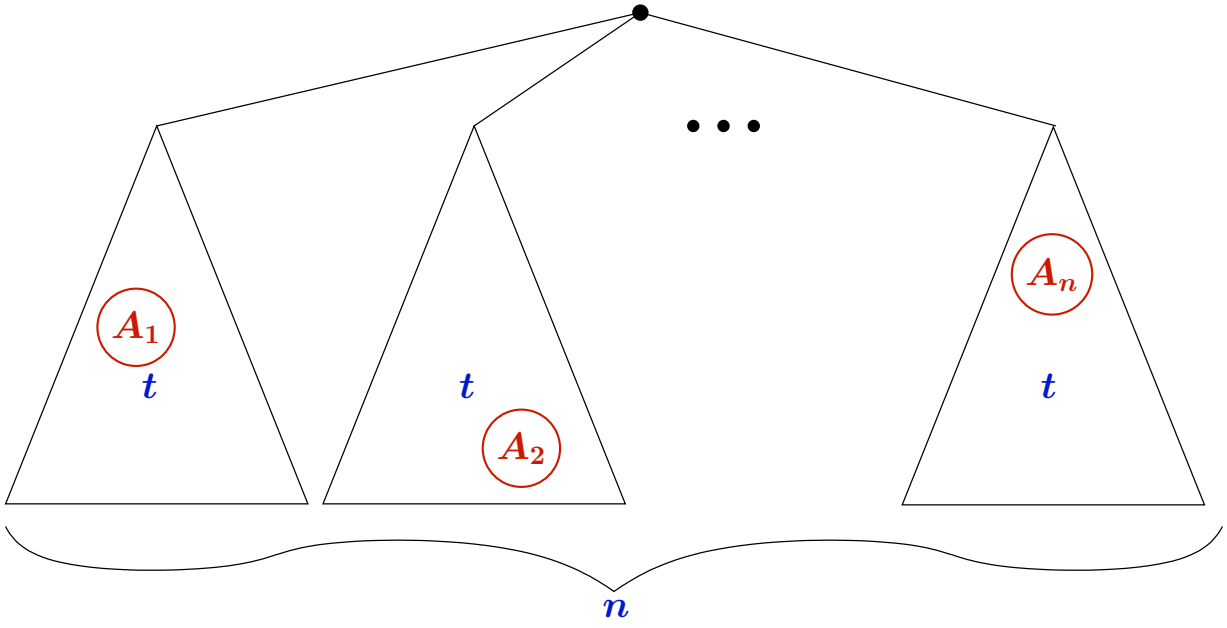

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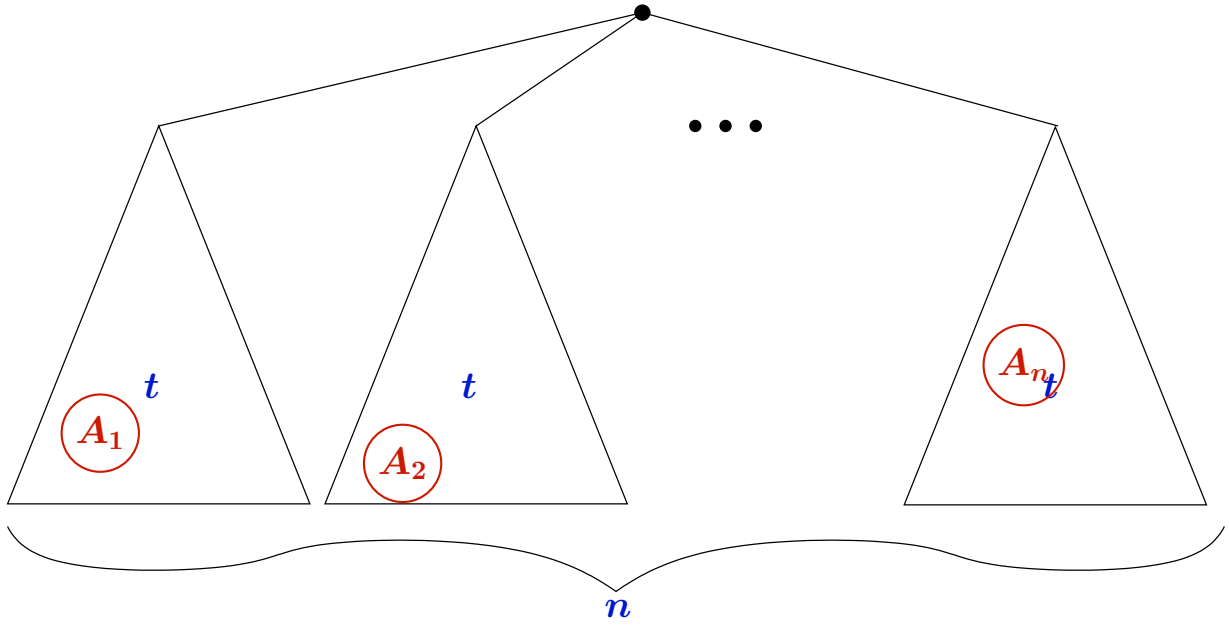

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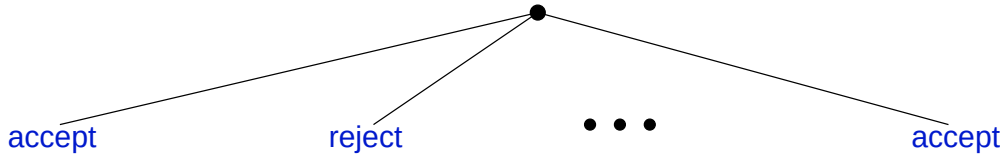

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What is Left to do?

- Quest for large tractable fragments
- Quest for large decidable fragments
- Settings where input and/or output schemas are fixed are also relevant in practice
- Building a complete typechecker
- Building a hybrid complete/incomplete typechecker